

§1.2-1.3 Practice WS

Name: Key

Find the following limits analytically.

1. Given $f(x) = x + 7$ and $g(x) = x^2$

a) $\lim_{x \rightarrow -3} f(x) = 4$

b) $\lim_{x \rightarrow 4} g(x) = 16$

c) $\lim_{x \rightarrow -3} g(f(x)) = 16$

2. Given $f(x) = -2x + 5$ and $g(x) = \sqrt{x}$, find the following limit.

$$\lim_{x \rightarrow -10} g(f(x)) = \lim_{x \rightarrow 25} \sqrt{x} = \sqrt{25} = 5$$

3. $\lim_{x \rightarrow 0} \frac{\sqrt{2+x} - \sqrt{2}}{x} \cdot \frac{\sqrt{2+x} + \sqrt{2}}{\sqrt{2+x} + \sqrt{2}} = \lim_{x \rightarrow 0} \frac{2+x-2}{x(\sqrt{2+x} + \sqrt{2})}$

$$= \lim_{x \rightarrow 0} \frac{x}{x(\sqrt{2+x} + \sqrt{2})} = \lim_{x \rightarrow 0} \frac{1}{\sqrt{2+x} + \sqrt{2}}$$

$$= \frac{1}{\sqrt{2} + \sqrt{2}} = \frac{1}{2\sqrt{2}} = \left(\frac{\sqrt{2}}{4}\right)$$

4. $\lim_{x \rightarrow 1} \frac{\sqrt{3-x} + \sqrt{3}}{x-1} = \frac{\sqrt{2} + \sqrt{3}}{0}$

DNE

*unbounded behavior

5. $\lim_{x \rightarrow 0} \frac{1 - \cos x}{x} = 0$

6. $\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$

7. $\lim_{x \rightarrow 0} \frac{(\sin x)(x+2)}{x} = \lim_{x \rightarrow 0} \frac{\sin x}{x} \cdot \lim_{x \rightarrow 0} x+2$

$$1 \cdot 2 = 2$$

8. $\lim_{x \rightarrow 0} \frac{5 - 5 \cos x}{3x} = \lim_{x \rightarrow 0} \frac{5}{3} \cdot \lim_{x \rightarrow 0} \frac{1 - \cos x}{x}$

$$= \frac{5}{3} \cdot 0 = 0$$

9. $\lim_{x \rightarrow 0} \frac{\tan x}{4x} = \lim_{x \rightarrow 0} \frac{\frac{\sin x}{\cos x}}{4x} = \lim_{x \rightarrow 0} \frac{\sin x}{4x \cos x}$

$$= \lim_{x \rightarrow 0} \frac{\sin x}{x} \cdot \lim_{x \rightarrow 0} \frac{1}{4 \cos x} = 1 \cdot \frac{1}{4} = \frac{1}{4}$$

10. $\lim_{x \rightarrow 2} \frac{x^2 + x - 6}{x^2 - 4} = \lim_{x \rightarrow 2} \frac{(x-2)(x+3)}{(x-2)(x+2)}$

$$= \lim_{x \rightarrow 2} \frac{x+3}{x+2} = \frac{5}{4}$$

11. $\lim_{x \rightarrow 2} \frac{\frac{1}{x} - \frac{1}{2}}{x-2} = \lim_{x \rightarrow 2} \frac{\frac{2-x}{2x}}{x-2} = \lim_{x \rightarrow 2} \frac{2-x}{2x(x-2)}$

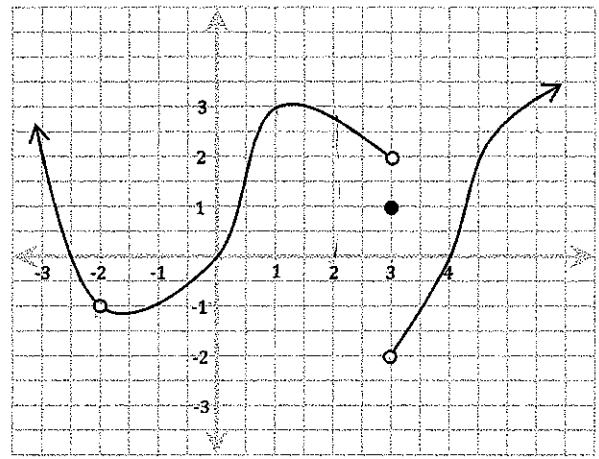
$$\lim_{x \rightarrow 2} \frac{-1(x-2)}{2x(x-2)} = \lim_{x \rightarrow 2} \frac{-1}{2x} = -\frac{1}{4}$$

12. $\lim_{x \rightarrow 0} \frac{(x-5)^2 - 25}{x} = \lim_{x \rightarrow 0} \frac{x^2 - 10x + 25 - 25}{x}$

$$= \lim_{x \rightarrow 0} \frac{x^2 - 10x}{x} = \lim_{x \rightarrow 0} \frac{x(x-10)}{x}$$

$$= \lim_{x \rightarrow 0} x - 10 = -10$$

Use the graph of $f(x)$ to evaluate the following, if possible.



13. $\lim_{x \rightarrow 1} f(x) = 3$

14. $\lim_{x \rightarrow 3^+} f(x) = -2$

15. $\lim_{x \rightarrow 3^-} f(x) = 2$

16. $\lim_{x \rightarrow 3} f(x) = \text{DNE}$

17. $f(3) = 1$

18. $\lim_{x \rightarrow -2^-} f(x) = -1$

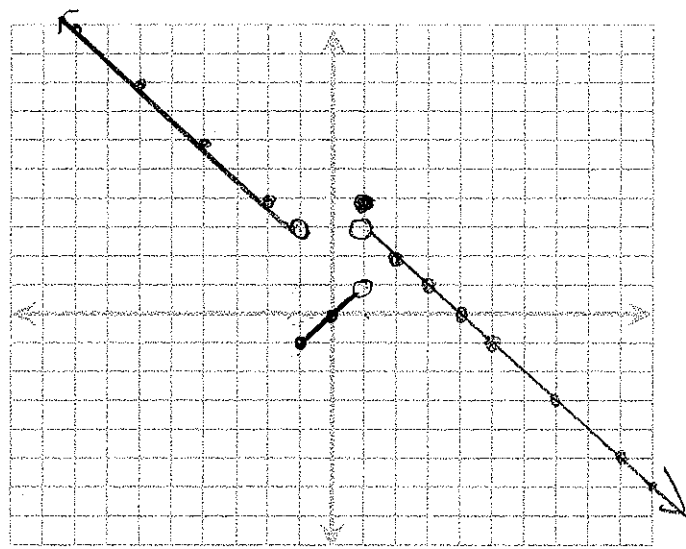
19. $\lim_{x \rightarrow -2^+} f(x) = -1$

20. $\lim_{x \rightarrow -2} f(x) = -1$

21. $f(-2) = \text{undefined}$

Sketch the graph of the function and answer the following questions.

22. $f(x) = \begin{cases} 2-x, & x < -1 \\ x, & -1 \leq x < 1 \\ 4, & x = 1 \\ 4-x, & x > 1 \end{cases}$



23. $\lim_{x \rightarrow -1^-} f(x) = 3$

24. $\lim_{x \rightarrow -1^+} f(x) = -1$

25. $\lim_{x \rightarrow 1} f(x) = \text{DNE}$

26. $\lim_{x \rightarrow 1^+} f(x) = 3$

27. $\lim_{x \rightarrow 1^-} f(x) = 1$

28. $\lim_{x \rightarrow 1} f(x) = \text{DNE}$

On a separate sheet of paper, sketch a graph of a function with the following properties.

29. $f(x)$ given:
 $f(-1) = 2$
 $f(0) = -1$
 $f(1) = 3$
 $\lim_{x \rightarrow 1} f(x)$ does not exist

30. $f(x)$ given:
 $f(x) = 1$, for $-2 \leq x \leq 1$
 $\lim_{x \rightarrow 1^+} f(x) = 3$
 $\lim_{x \rightarrow -2} f(x) = 1$

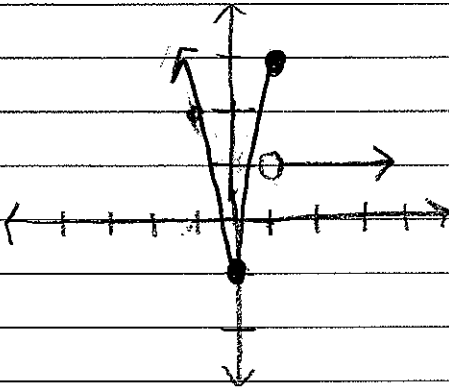
31. $f(x)$ given:
 $f(0) = 1$
 $\lim_{x \rightarrow 0^-} f(x) = 2$
 $\lim_{x \rightarrow 0^+} f(x) = 3$

32. $f(x)$ given:
 $\lim_{x \rightarrow 0} f(x) = -2$
 $f(0) = 1$
 $f(2) = 3$
 $\lim_{x \rightarrow 2} f(x)$ does not exist

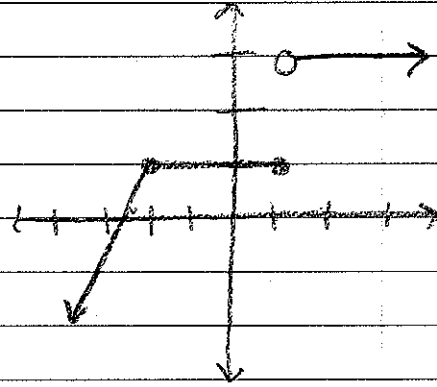
1.2-1.3 Practice

Sketch a graph with the given properties.

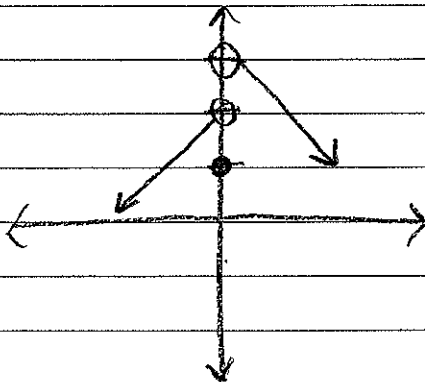
(29) $f(-1) = 2$
 $f(0) = -1$
 $f(1) = 3$
 $\lim_{x \rightarrow 1} f(x)$ DNE



(30) $f(x) = 1$ for $-2 \leq x \leq 1$
 $\lim_{x \rightarrow 1^+} f(x) = 3$
 $\lim_{x \rightarrow -2} f(x) = 1$



(31) $f(0) = 1$
 $\lim_{x \rightarrow 0^-} f(x) = 2$
 $\lim_{x \rightarrow 0^+} f(x) = 3$



(32) $\lim_{x \rightarrow 0} f(x) = -2$
 $f(0) = 1$
 $f(2) = 3$
 $\lim_{x \rightarrow 2} f(x)$ DNE

