Section 1.2: Finding Limits Graphically + Numerically

$$\lim_{x \to c} f(x) = L$$

* Read "the limit as x approaches c of f(x) is L"

* The values of f(x) (y-values) get arbitraily close to a single number L as x approaches the target value c, but not equal to c.

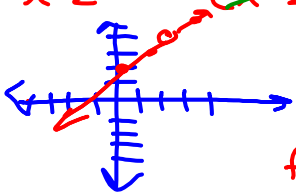
* the limit is a y-value

* not important what the value of the function equals at target value

Ex. 1 $\lim_{x \to 2} \frac{x^2-4}{x-2} = 4$

$$\frac{x^2-4}{x-2} = \frac{\cancel{(x-2)}(x+2)}{\cancel{(x-2)}} = x+2$$

hole @ (2,4)



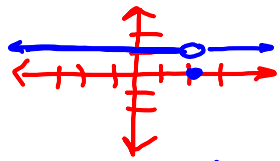
Numerically

x	1.9	1.99	1.999	2.001	2.01	2.1
f(x)	3.9	3.99	3.999	4.001	4.01	4.1

$$\lim_{x \to 2^-} \frac{x^2-4}{x-2} = 4 \qquad \lim_{x \to 2^+} \frac{x^2-4}{x-2} = 4$$

Ex. 2

Find $\lim_{x \to 2} f(x)$, when $f(x) = \begin{cases} 1, & x \neq 2 \\ 0, & x = 2 \end{cases}$



$\lim_{x \to 2} f(x) = 1$ $\qquad f(2) = 0$

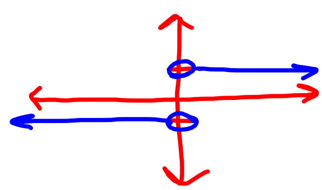
Numerically

x	1.9	1.99	1.999	2.001	2.01	2.1
$f(x)$	1	1	1	1	1	1

Limits that fail to exist

① Behavior differs from the left and right

$\lim_{x \to 0} \frac{|x|}{x}$

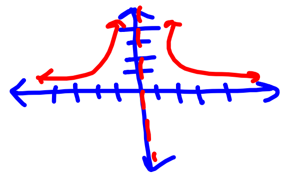


$\lim_{x \to 0^-} \frac{|x|}{x} = -1$ $\qquad \lim_{x \to 0^+} \frac{|x|}{x} = 1$

$\lim_{x \to 0} \frac{|x|}{x}$ DNE

(2) Unbounded behavior: There is a vertical asymptote at the target value

$\lim_{x \to 0} \frac{1}{x^4}$



The limit DNE

(3.) Oscillating Behavior: the y-values oscillate between two fixed values as $x \to c$

$\lim_{x \to 0} \sin\left(\frac{1}{x}\right)$