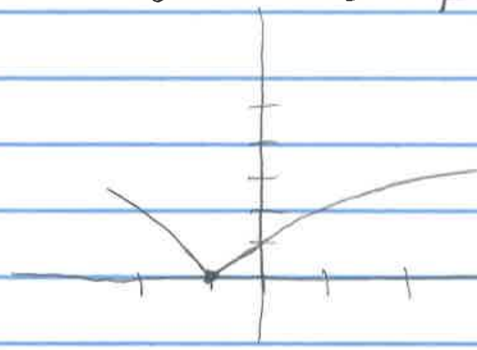
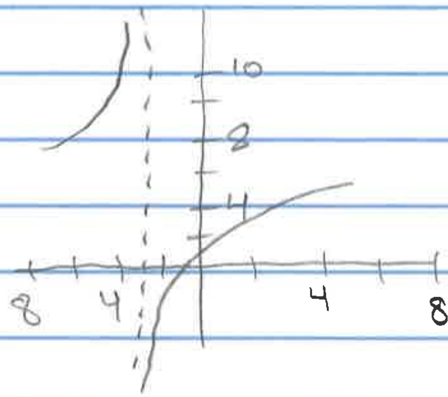
Chapter 2 Review

5) where is $f(x)$ differentiable?



differentiable everywhere except $x=-1$

7)



dif. everywhere except $x=-3$

9) Find the slope of the tangent line at the point

$f(x) = \frac{2}{3}x^2 - \frac{x}{6} \qquad (-1, 5/6)$

$f'(x) = \frac{4}{3}x - \left(\frac{6(1) - x(0)}{36}\right) = \frac{4}{3}x - \frac{6}{36}$

$f'(x) = \frac{4}{3}x - \frac{1}{6} \qquad f'(-1) = \frac{4}{3}(-1) - \frac{1}{6}$

$f'(-1) = -\frac{3}{2}$

Find the equation of the tangent line at the given point

11) $f(x) = x^3 - 1 \quad (-1, -2)$

$f'(x) = 3x^2$

$f'(x) = 3(-1)^2 = 3(1) = 3$

$y + 2 = 3(x + 1)$ or $y = 3x + 1$

15) $y = 25$

$y' = 0$

17) $f(x) = x^8 \quad f'(x) = 8x^7$

19) $h(t) = 3t^4 \quad h'(t) = 12t^3$

21) $f(x) = x^3 - 3x^2 \quad f'(x) = 3x^2 - 6x$

23) $h(x) = 6\sqrt{x} + 3\sqrt[3]{x}$

$h(x) = 6x^{\frac{1}{2}} + 3x^{1/3}$

$h'(x) = 3x^{-\frac{1}{2}} + x^{-2/3}$

$= \dfrac{3}{\sqrt{x}} + \dfrac{1}{x^{2/3}}$

25) $g(t) = \dfrac{2}{3t^2} = \dfrac{2}{3}t^{-2}$

$g'(t) = -\dfrac{4}{3}t^{-3} = \dfrac{-4}{3t^3}$

29) $f(\theta) = 3\cos\theta - \dfrac{\sin\theta}{4}$

$f'(\theta) = -3\sin\theta - \left(\dfrac{4(\cos\theta) - \sin\theta(0)}{16}\right)$

$f'(\theta) = -3\sin\theta - \dfrac{4\cos\theta}{16}$

$f'(\theta) = -3\sin\theta - \dfrac{\cos\theta}{4}$

41) $f(x) = (3x^2+7)(x^2-2x+3)$

Product Rule

$f'(x) = (3x^2+7)(2x-2) + (x^2-2x+3)(6x)$

$f'(x) = 6x^3 - 6x^2 + 14x - 14 + 6x^3 - 12x^2 + 18x$

$$\boxed{\begin{aligned} f'(x) &= 12x^3 - 18x^2 + 32x - 14 \\ f'(x) &= 2(6x^3 - 9x^2 + 16x - 7) \end{aligned}}$$

Either one is ok!

43) $h(x) = \sqrt{x} \cdot \sin x$

$h(x) = x^{\frac{1}{2}} \sin x$

$h'(x) = x^{\frac{1}{2}} \cdot \cos x + \sin x \cdot \frac{1}{2} x^{-\frac{1}{2}}$

$$\boxed{h'(x) = \sqrt{x}\, \cos x + \frac{\sin x}{2\sqrt{x}}}$$

45) $f(x) = \dfrac{x^2 + x - 1}{x^2 - 1}$ $\qquad \dfrac{lo \cdot dhi - hi \cdot dlo}{lo^2}$

Quotient: $f'(x) = \dfrac{(x^2-1)(2x+1) - (x^2+x-1)(2x)}{(x^2-1)^2}$

$f'(x) = \dfrac{2x^3 + x^2 - 2x - 1 - (2x^3 + 2x^2 - 2x)}{(x^2-1)^2}$

$f'(x) = \dfrac{\cancel{2x^3} + x^2 - 2x - 1 - \cancel{2x^3} - 2x^2 + 2x}{(x^2-1)^2}$

$f'(x) = \dfrac{-x^2 - 1}{(x^2-1)^2} = \boxed{\dfrac{-(x^2+1)}{(x^2+1)^2}}$

47) $f(x) = \dfrac{1}{4-3x^2} = (4-3x^2)^{-1}$ Use Product here!

$f'(x) = -(4-3x^2)^{-2} \cdot -6x$

$$\boxed{f'(x) = \frac{6x}{(4-3x^2)^2}}$$

49) $y = \dfrac{x^2}{\cos x}$

$y' = \dfrac{\cos x \cdot 2x - x^2(-\sin x)}{(\cos x)^2}$

$\boxed{y' = \dfrac{2x\cos x + x^2 \sin x}{\cos^2 x}}$

Product Rule!

51) $y = 3x^2 \cdot \sec x$ ← Remember all Trig derivatives!

$y' = 3x^2(\sec x \tan x) + \sec x(6x)$

$\boxed{y' = 3x^2 \sec x \tan x + 6x \sec x}$

Product ↓

53) $y = x\cos x - \sin x$

$y' = x(-\sin x) + \cos x\,(1) - \cos x$

$y' = -x\sin x + \cos x - \cos x$

$\boxed{y' = -x\sin x}$

55) Find the equation of the tangent line at the point

$f(x) = \dfrac{2x^3 - 1}{x^2} \quad (1,1) \rightarrow f(x) = (2x^3 - 1)x^{-2}$

$f(x) = 2x - x^{-2}$

$f'(x) = 2 + 2x^{-3} = 2 + \dfrac{2}{x^3}$

$f'(1) = 2 + \dfrac{2}{1} = 4 \rightarrow \boxed{y - 1 = 4(x-1)}$

57) $f(x) = -x\tan x \qquad (0,0)$

$f'(x) = -x\sec^2 x + \tan x \cdot (-1)$

$f'(x) = -x\sec^2 x - \tan x$

$f'(0) = -0\sec^2(0) - \tan(0)$

$f'(0) = 0 - 0$

$f'(0) = 0$ tangent line

$y - 0 = 0(x-0)$

$\boxed{y = 0}$

59) velocity of an object in meters/second is $v(t) = 36 - t^2 \quad 0 \le t \le 6$. Find the velocity and acceleration at $t = 4$ seconds.

Velocity $\rightarrow$ $V(4) = 36 - (4)^2$ $\quad \boxed{V(t) = 20 \text{ m/sec}}$

$= 36 - 16$

Acceleration $\rightarrow$ $v'(t) = a(t)$

$v'(t) = -2t = a(t)$

$a(4) = -2(4) = \boxed{-8 \text{ m/sec}^2}$

79) $f(x) = \sqrt{1-x^3}$ find the derivative at $(-2,3)$

$f(x) = (1-x^3)^{1/2}$

$f'(x) = \frac{1}{2}(1-x^3)^{-\frac{1}{2}} \cdot (-3x^2)$

$F'(x) = \dfrac{-3x^2}{2\sqrt{1-x^3}} \qquad f'(-2) = \dfrac{-3(-2)^2}{2\sqrt{1-(-2)^3}} = \dfrac{-12}{2(3)} = \boxed{-2}$

Double chain Rule

$f(x) = \cos^2(x^2)$
$f(x) = ((\cos(x^2))^2$

$f'(x) = 2(\cos(x^2))^1 \cdot (-\sin(x^2) \cdot 2x)$

$\boxed{f'(x) = -4x\cos(x^2)\sin(x^2)}$

$f(x) = \tan^2 x = (\tan x)^2$
$f'(x) = 2(\tan x)^1 \cdot \sec^2 x$
$\boxed{f'(x) = 2\tan x\sec^2 x}$

$f(x) = 5\cos^2(\pi x)$

$f(x) = 5(\cos \pi x)^2$

$f'(x) = 10(\cos \pi x)^1 \cdot (-\sin(\pi x)) \cdot (\pi)$

$\boxed{f'(x) = -10\pi\cos(\pi x)\sin(\pi x)}$

Related Rates

the radius, r, of a sphere is increasing at a rate of 2 in/sec

find the rate of change for the volume when r = 6

$$V = \frac{4}{3}\pi r^3 \rightarrow \frac{dV}{dt} = 4\pi r^2 \cdot \frac{dr}{dt}$$

$$\frac{dV}{dt} = 4\pi(6)^2 \cdot (2)$$

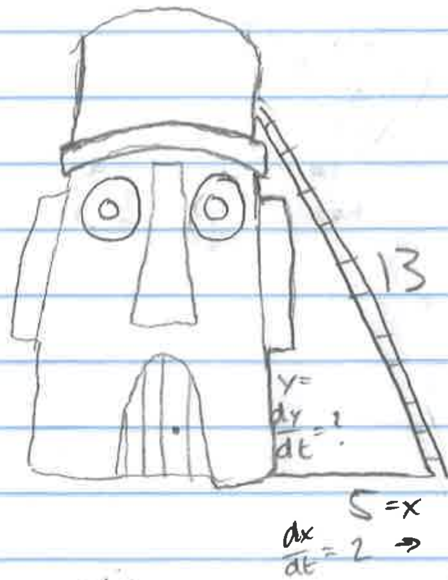
$$\boxed{\frac{dV}{dt} = 288\pi \text{ in}^3/\text{sec}}$$

the radius of a circle is increasing at a rate of 3 cm/min. Find the rate of change of the area when r = 24

$$A = \pi r^2 \rightarrow \frac{dA}{dt} = 2\pi r \cdot \frac{dr}{dt}$$

$$\frac{dA}{dt} = 2\pi(24)\cdot(3) = \boxed{144\pi \text{ cm}^2/\text{min}}$$

Spongebob is trying to annoy Squidward and leans a 13 foot ladder against his house. Patrick starts to pull the ladder away at 2ft/sec. How fast is Spongebob (the top of the ladder) falling when its 5 feet from the house.



first find y with Pythagorean's theorem.

$$13^2 = 5^2 + y^2$$

$$169 = 25 + y^2$$

$$\sqrt{144} = \sqrt{y^2}$$

$$y = 12$$

then. → $x^2 + y^2 = 13^2$

take derivative $2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$

$x = 5 \quad y = 12$

$\frac{dx}{dt} = 2$

$$2(5)(2) + 2(12)\frac{dy}{dt} = 0$$

$$20 + 24\frac{dy}{dt} = 0$$

$$24\frac{dy}{dt} = -20$$

$$\boxed{\frac{dy}{dt} = \frac{-20}{24} = -\frac{5}{6} \text{ ft/sec}}$$