Day 1 3.7 HW #3 - 11 odd

#3) The Sum is S and the product is a minimum.

Maximize Product

Primary:

$P = x(y)$

Secondary

$x + y = S$

$y = (S - x)$

$y = \frac{2}{2}\left(\frac{S}{1}\right) - \left(\frac{S}{2}\right)$

$y = \frac{2S - S}{2} = \frac{S}{2}$

$P = x(S - x)$

$P = Sx - x^2$

$P' = S - 2x = 0$

$S = 2x$

$\frac{S}{2} = x$

the Product is a max when $x = y = \frac{S}{2}$

Note*

If you want to use the Second derivative test, here is what you do.

1. take the second derivative

$P'' = -2$

if $P'' > 0$ then it is a minimum
if $P'' < 0$ then it is a maximum

$P'' = -2 < 0$ ← this proves that $\frac{S}{2}$ is a maximum

#5) the product is 192 and the sum of the first plus three times the second is a minimum.

$\boxed{\text{Minimize Sum}}$

Secondary: $xy = 192$

$y = \dfrac{192}{x}$

Primary:

$P = x + 3y$

$P = x + 3\left(\dfrac{192}{x}\right)$

$P = x + \dfrac{576}{x} \rightarrow x + 576x^{-1}$

$P' = 1 - 576x^{-2} \rightarrow 1 - \dfrac{576}{x^2} = 0$

$x^2 \cdot 1 = \dfrac{576}{\cancel{x^2}} \cdot \cancel{x^2} \rightarrow \sqrt{x^2} = \sqrt{576}$

$\downarrow$ *test ↗

$\cancel{x = \pm 24}$

$x = 24$

$y = \dfrac{192}{24} = 8$

$\boxed{\text{The Sum is a min when } x = 24 \text{ and } y = 8}$

first derivative test

$+ \quad - \quad +$

$\cancel{-24}$ ~~max~~ $\qquad$ 24 min

Second derivative test

$P'' = 1152x^{-3} = \dfrac{1152}{x^3}$

plug in ± 24

$P''(-24) < 0 \leftarrow$ max

$\boxed{P''(24) > 0 \leftarrow \text{min}}$

*Pick whatever test you are more comfortable with! One or the other. No need for both

#7) The sum of the first and twice the second is 100. and the product is a maximum.

$\boxed{\text{Maximize Product}}$

Primary

$xy = P$

Secondary:

$x + 2y = 100$

$x = 100 - 2y$

$P = (100 - 2y)y$

$P = 100y - 2y^2$

$P' = 100 - 4y = 0$

$100 = 4y$

$25 = y$

$x = 100 - 2(25)$

$x = 100 - 50$

$x = 50$

FDT

$+ \quad -$

25

max ✓

SDT

$P'' = -4 < 0$

max

$\boxed{\text{Product is a max when } x = 50 \text{ and } y = 25}$

#9) Perimeter : 100 meter

find the length and width of a rectangle that has the given perimeter and a maximum area,

$\boxed{\text{Maximize Area}}$

Primary

$A = x\boxed{y}$

Secondary

$2x + 2y = 100$

$2y = 100 - 2x$

$\boxed{y = 50 - x}$

$A = x(50 - x)$

$A = 50x - x^2$

$A' = 50 - 2x = 0$

$50 = 2x$

$25 = x$

$y = 50 - (25)$

$y = 25$

FDT

+ | −

25 max ✓

SDT

$A'' = -2 < 0$

max

Area is a max when

$x = y = 25$

#11) Area = 64 ft^2

Find the length and width of a rectangle that has the given area and a minimum perimeter

$$\boxed{\text{Minimize Perimeter}}$$

Primary

$P = 2x + 2y$

$P = 2x + 2\left(\frac{64}{x}\right)$

$P = 2x + \frac{128}{x}$ or $P = 2x + 128x^{-1}$

$P' = 2 - \frac{128}{x^2} = 0$

$x^2 \cdot 2 = \frac{128}{\cancel{x^2}} \cancel{x^2}$

$2x^2 = 128$

$x^2 = 64$

$\cancel{x = \pm 8}$ test →

$x = 8$

Secondary

$A = xy$

$64 = xy$

$y = \frac{64}{x}$

$y = \frac{64}{8}$

$y = 8$

FDT

$+ \quad - \quad +$

~~−8 max~~ 8 min ✓

SDT

$P'' = \frac{256}{x^3}$

$P''(-8) < 0 \leftarrow$ max

$\boxed{P''(8) > 0 \leftarrow \text{min ✓}}$

$$\boxed{\text{Perimeter is a min when } x = y = 8}$$