

#3) The sum is  $S$  and the product is a minimum.

Maximize Product

Primary:

$$P = x(y)$$

Secondary

$$x + y = S$$

$$y = \frac{2}{2} \left( \frac{S}{2} \right) - \left( \frac{S}{2} \right)$$

$$y = S - x$$

$$y = \frac{2S - S}{2} = \frac{S}{2}$$

$$P = x(S - x)$$

$$P = Sx - x^2$$

$$P' = S - 2x = 0$$

$$S = 2x$$

$$\frac{S}{2} = x$$

Note\*

IF you want to use the second derivative test, here is what you do.

① take the second derivative

$$P'' = -2$$

if  $P'' > 0$  then it is a minimum  
if  $P'' < 0$  then it is a maximum

$P'' = -2 < 0$  ← this proves that  $\frac{S}{2}$  is a maximum

The Product is a max when  $x = y = \frac{S}{2}$



#5) The product is 192 and the sum of the first plus three times the second is a minimum.

Minimize Sum

Primary:

$$P = x + 3y$$

$$P = x + 3\left(\frac{192}{x}\right)$$

$$P = x + \frac{576}{x} \rightarrow x + 576x^{-1}$$

$$P' = 1 - 576x^{-2} \rightarrow 1 - \frac{576}{x^2} = 0$$

$$x^2 \cdot 1 = \frac{576}{x^2} \cdot x^2 \rightarrow \sqrt{x^2} = \sqrt{576}$$

↓ \*test →

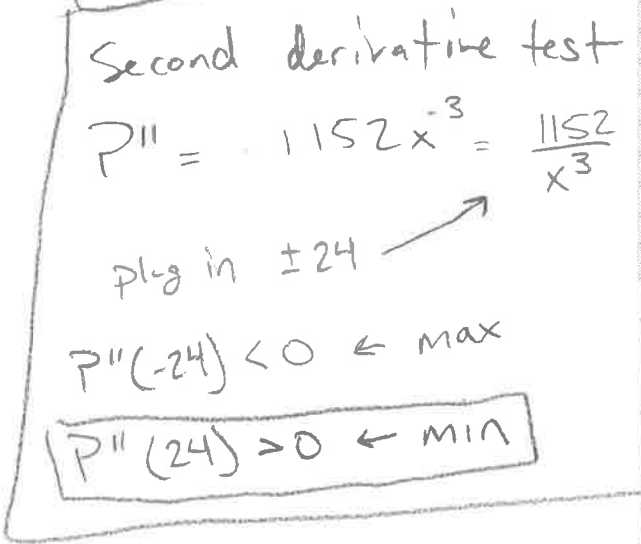
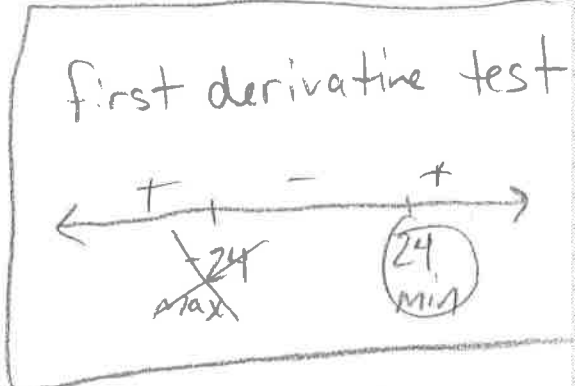
$$x = \pm 24$$
$$x = 24$$

$$y = \frac{192}{24} = 8$$

The sum is a min when  $x = 24$  and  $y = 8$

Secondary:  $xy = 192$

$$y = \frac{192}{x}$$



\*Pick whatever test you are more comfortable with! One or the other. No need for both



#7) The sum of the first and twice the second is 100.  
and the product is a maximum.

### Maximize Product

Primary  
 $XY = P$

Secondary:  
 $X + 2y = 100$

$$X = 100 - 2y$$

$$P = (100 - 2y)y$$

$$P = 100y - 2y^2$$

$$P' = 100 - 4y = 0$$

$$100 = 4y$$

$$25 = y$$

$$X = 100 - 2(25)$$

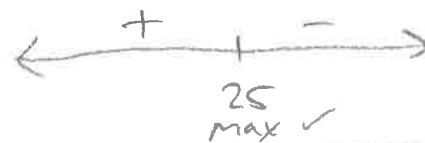
$$X = 100 - 50$$

$$X = 50$$

Product is a max when

$$X = 50 \text{ and } y = 25$$

FDT



SDT

$$P'' = -4 < 0$$

↑  
max



#9) Perimeter: 100 meter

find the length and width of a rectangle that has the given perimeter and a maximum area,

Maximize Area

Primary  
 $A = XY$

Secondary  
 $2X + 2Y = 100$

$$A = X(50 - X)$$

$$2Y = 100 - 2X$$
$$Y = 50 - X$$

$$A = 50X - X^2$$

$$A' = 50 - 2X = 0$$

$$50 = 2X$$

$$25 = X$$

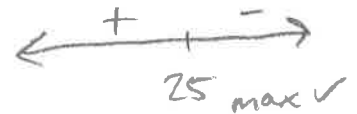
$$Y = 50 - (25)$$

$$Y = 25$$

Area is a max when

$$X = Y = 25$$

FDT



SDT

$$A'' = -2 < 0$$

↑  
max





#11) Area =  $64 \text{ ft}^2$

Find the length and width of a rectangle that has the given area and a minimum perimeter

**Minimize Perimeter**

Primary

$$P = 2x + 2y$$

$$P = 2x + 2\left(\frac{64}{x}\right)$$

$$P = 2x + \frac{128}{x} \text{ or } P = 2x + 128x^{-1}$$

$$P' = 2 - \frac{128}{x^2} = 0$$

$$x^2 \cdot 2 = \frac{128}{x^2} \cdot x^2$$

$$2x^2 = 128$$

$$x^2 = 64$$

$$x = \pm 8 \text{ test} \rightarrow$$

$$x = 8$$

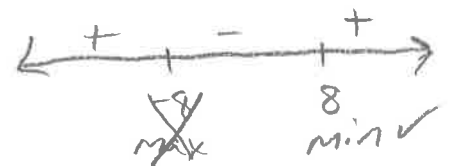
Secondary

$$A = xy$$

$$64 = xy$$

$$y = \frac{64}{x}$$

FDT



SDT

$$P'' = \frac{256}{x^3}$$

$$P''(-8) < 0 \leftarrow \text{max}$$

$$P''(8) > 0 \leftarrow \text{min} \checkmark$$

**Perimeter is a min when  $x = y = 8$**

