Calculus Name Key
Power, Product, and Quotient Rule Practice

For each of the following, determine if the easiest way to find the derivative: power, product, or quotient rule. If the easiest way is to rewrite the function to use the power rule, then rewrite the function. Find the derivative of each function.

1. $y=2x^{-4}(x+6)$ *Rewrite + power

$y=2x^{-3}+12x^{-4}$

$y'=-6x^{-4}-48x^{-5}$

$\boxed{y'=\frac{-6}{x^4}-\frac{48}{x^5}}$

2. $y=\frac{\cancel{\csc x}\ \sec x}{x^5}$ *Quotient

$y'=\frac{x^5\sec x\tan x-5x^4\sec x}{x^{10}}$

$y'=\frac{x\sec x\tan x-5\sec x}{x^6}$

3. $y=\frac{2}{x^3}-4x+\frac{1}{5\sqrt[4]{x}}$ *Rewrite and power

$y=2x^{-3}-4x+\frac{1}{5}x^{-1/4}$

$y'=-6x^{-4}-4-\frac{1}{20}x^{-5/4}$

$y'=\frac{-6}{x^4}-4-\frac{1}{20x^{5/4}}$

4. $y=\frac{3x^5-6x^7+x-2}{x^4}$ *Rewrite + power

$y=3x-6x^3+x^{-3}-2x^{-4}$

$y'=3-18x^2-3x^{-4}+8x^{-5}$

$y'=3-18x^2-\frac{3}{x^4}+\frac{8}{x^5}$

5. $y=\cot x\,\cancel{\sec x}\ \csc x$ *product

$y'=-\cot x\csc x\cot x+\csc x(-\csc^2 x)$

$y'=-\cot^2 x\csc x-\csc^3 x$

6. $y=\frac{5}{\sqrt[10]{x^1}}$ *Rewrite + power

$y=5x^{-1/10}$

$y'=\frac{-1}{2}x^{-11/10}$

$y'=\frac{-1}{2x^{11/10}}$

7. $y=\frac{4x^{\frac{2}{5}}}{x^2}$ *Rewrite and Power

$y=4x^{-8/5}$

$y'=\frac{-32}{5}x^{-13/5}$

$y'=\frac{-32}{5x^{13/5}}$

8. $y=(2x-x^2)(x+1)$ *Rewrite and Power

$y=2x^2+2x-x^3-x^2=-x^3+x^2+2x$

$y=x^2+2x-x^3$

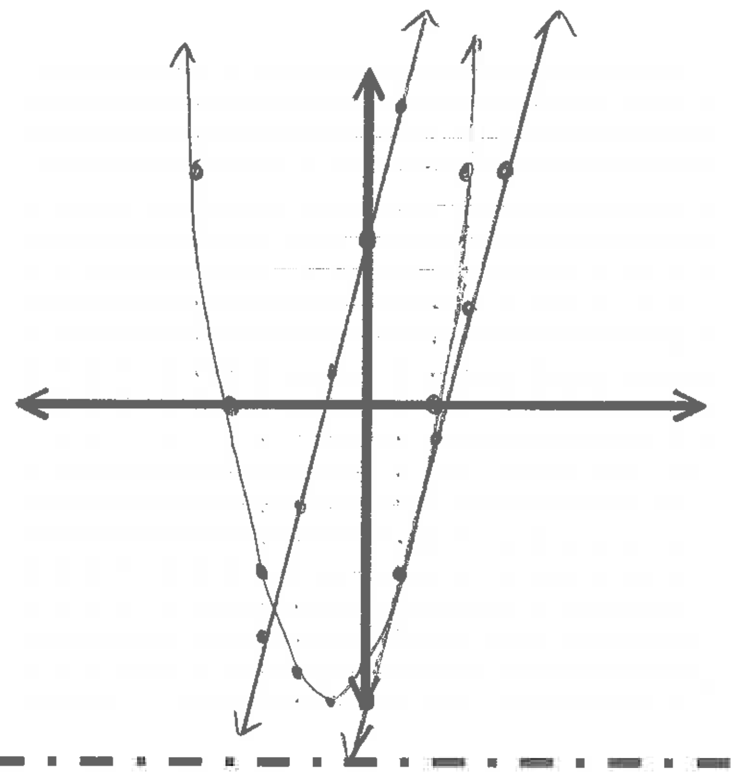
$y'=-3x^2+2x+2$

Calculus
2.2 Practice

*same as rates of change worst

Name Key

1. Graph $f(x) = x^2 + 2x - 8$ and $g(x) = 4x + 5$.



2. Find the derivative of *f(x)*.

$f'(x) = 2x + 2$

3. Find the coordinate(s) of any points that have a tangent line that is parallel to *g(x)*.

$2x + 2 = 4$
$2x = 2$
$x = 1$

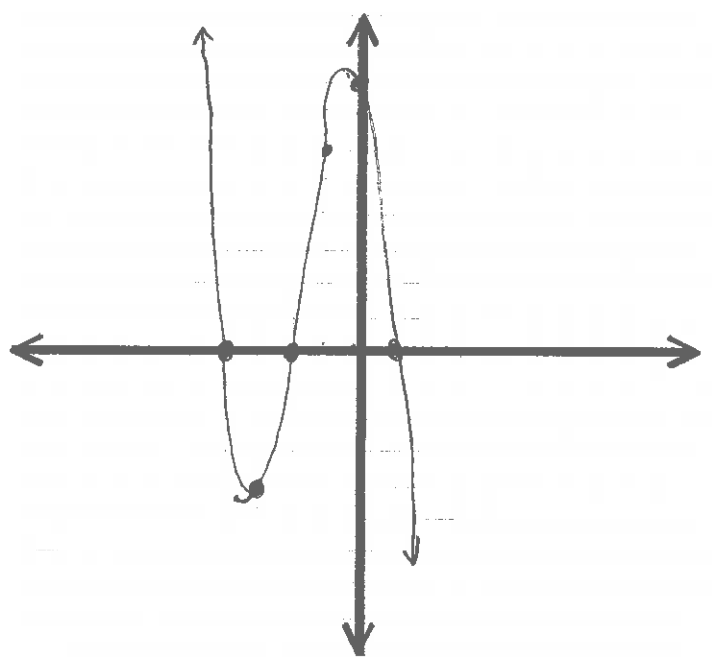
$(1, -5)$

4. Write the equation of the tangent line parallel to *g(x)*.

$y + 5 = 4(x - 1)$

5. Graph the tangent line.

6. Graph $g(x) = -x^3 - 5x^2 - 2x + 8$.



7. Use the calculator to determine the coordinates of any relative maximums or minimums.

Min $(-3.12, -4.06)$
Max $(-0.21, 8.21)$

8. Find the derivative of *g(x)*.

$g'(x) = -3x^2 - 10x - 2$

9. What type of lines will be tangent at the relative maximums and minimums? What will the slope of these lines be?

Horizontal $m = 0$

10. Algebraically determine the coordinates of the points that have horizontal tangents.

$-3x^2 - 10x - 2 = 0$

$x = \frac{10 \pm \sqrt{100 - 4(-3)(-2)}}{2(-3)}$

$x = \frac{10 \pm \sqrt{76}}{-6}$ $x \approx -3.12, -0.12$

$(-3.12, -4.06)$
$(-0.21, 8.21)$

11. Determine the coordinates any point(s) that $y = x\sqrt{3} + 2\cos x$ has a horizontal tangent on the interval $[0, 2\pi]$.

$y' = \sqrt{3} - 2\sin x$

$0 = \sqrt{3} - 2\sin x$

$\frac{\sqrt{3}}{2} = \sin x$

$x = \frac{\pi}{3}, \frac{2\pi}{3}$

$\left(\frac{\pi}{3}, \frac{\pi\sqrt{3}+3}{3}\right)$ $\left(\frac{2\pi}{3}, \frac{2\pi\sqrt{3}-3}{3}\right)$

12. a. Find $f''(x)$ (the second derivative: the derivative of the derivative) of $f(x) = x^4 + 3x^3 - 2x^2 + 8x - 9$.

$f'(x) = 4x^3 + 9x^2 - 4x + 8$

$f''(x) = 12x^2 + 18x - 4$

b. Find $f'''(x)$

$f'''(x) = 24x + 18$

Average Velocity can be found by: $\text{Avg Velocity} = \frac{\text{Change in distance}}{\text{Change in time}} = \frac{\Delta s}{\Delta t}$

13. A ball is dropped from a height of 100 ft, its height s at time t is given by the position function $s = -16t^2 + 100$, where s is measured in feet and t is measured in seconds. Find the average velocity over the following intervals.

a. $[1,2]$ $(1, 84)$ $(2, 36)$

$\frac{\Delta s}{\Delta t} = \frac{84-36}{2-1} = -48$ ft/sec

b. $[1,1.5]$ $(1, 84)$ $(1.5, 64)$

$\frac{\Delta s}{\Delta t} = \frac{84-64}{1.5-1} = -40$ ft/sec

b. If a velocity is negative, what does that tell us?

The object is falling

The velocity function is the derivative of the position function. Speed is the absolute value of velocity.

14. At t=0, a diver jumps from a diving board 32 feet above the water. The position of the diver is given by $s(t) = -16t^2 + 16t + 32$ where s is measured in feet and t in seconds.

a. When does the diver hit the water?

$0 = -16t^2 + 16t + 32$

$0 = t^2 - t - 2$

$(t-2)(t+1)$

$t = 2$ $t = -1$

2 seconds

b. What is the diver's velocity at impact?

$v(t) = -32t + 16$

$v(2) = -32(2) + 16 = -48$ ft/sec

15. A projectile is shot upward 200m with an initial velocity of 120 meters per second. What is its velocity after 5 seconds given its position function is $s(t) = -4.9t^2 + v_0 t + s_0$.

$S(t) = -4.9t^2 + 120t + 200$

$v(t) = -9.8t + 120$

$v(5) = -9.8(5) + 120 = 71$ m/sec