Section 2.6

(1.) Given $y = \sqrt{x}$

$y = x^{\frac{1}{2}}$

$\frac{dy}{dt} = \frac{1}{2} \cdot x^{-1/2} \cdot \frac{dx}{dt}$

$\frac{dy}{dt} = \frac{1}{2x^{1/2}} \left(\frac{dx}{dt}\right)$

(a.) Find: $\frac{dy}{dt}$ when $x = 4$

Given: $\frac{dx}{dt} = 3$

$\frac{dy}{dt} = \frac{1}{2(4)^{1/2}}(3)$

$\boxed{\frac{dy}{dt} = \frac{3}{4}}$

(b.) Find: $\frac{dx}{dt}$ when $x = 25$

Given: $\frac{dy}{dt}$ when $\frac{dy}{dt} = 2$

$2 = \frac{1}{2(25)^{1/2}}\left(\frac{dx}{dt}\right)$

$2 = \frac{1}{10}\left(\frac{dx}{dt}\right)$

$\boxed{20 = \frac{dx}{dt}}$

(2.) Given $y = 2(x^2 - 3x)$

$y = 2x^2 - 6x$

$\frac{dy}{dt} = 4x\left(\frac{dx}{dt}\right) - 6\left(\frac{dx}{dt}\right)$

$\frac{dy}{dt} = \frac{dx}{dt}(4x - 6)$

(a) Find: $\frac{dy}{dt}$ when $x = 3$

Given: $\frac{dx}{dt} = 2$

$\frac{dy}{dt} = 2[4(3) - 6]$

$\boxed{\frac{dy}{dt} = 12}$

(b) Find: $\frac{dx}{dt}$ when $x = 1$

Given: $\frac{dy}{dt} = 5$

$5 = \frac{dx}{dt}[4(1) - 6]$

$5 = \frac{dx}{dt}(-2)$

$\boxed{\frac{-5}{2} = \frac{dx}{dt}}$

③ Given: $xy = 4$

$x\left(\frac{dy}{dt}\right) + y\left(\frac{dx}{dt}\right) = 0$ or $x\left(\frac{dy}{dt}\right) = -y\left(\frac{dy}{dt}\right)$

ⓐ Find $\frac{dy}{dt}$ when $x = 8$

Given: $\frac{dx}{dt} = 10$

* Need y when $x = 8$

$8y = 4$

$y = \frac{1}{2}$

$8\left(\frac{dy}{dt}\right) = -\frac{1}{2}(10)$

$8\left(\frac{dy}{dt}\right) = -5$

$\boxed{\frac{dy}{dt} = \frac{-5}{8}}$

ⓑ Find $\frac{dx}{dt}$ when $x = 1$

Given: $\frac{dy}{dt} = -6$

* Need y when $x = 1$

$(1)(y) = 4$

$y = 4$

$(1)(-6) = -4\left(\frac{dy}{dt}\right)$

$\frac{6}{4} = \frac{dy}{dt}$

$\boxed{\frac{3}{2} = \frac{dy}{dt}}$

⑤ Given

$y = x^2 + 1$

$\frac{dx}{dt} = 2\ cm/sec$

Find

dy/dt when $x = 1$

$\frac{dy}{dt} = 2x\left(\frac{dx}{dt}\right)$

$\frac{dy}{dt} = 2(1)(2)$

$\boxed{\frac{dy}{dt} = 4\ cm/sec}$

(6) Given

$y = \frac{1}{1+x^2}$

$\frac{dx}{dt} = 2$ cm/sec

Find

$\frac{dy}{dt}$ when $x = -2$

$y = (1+x^2)^{-1}$

$\frac{dy}{dt} = -1(1+x^2)^{-2}(2x\frac{dx}{dt})$

$\frac{dy}{dt} = \frac{-2x}{(1+x^2)^2}\left(\frac{dx}{dt}\right)$

$\frac{dy}{dt} = \frac{-2(-2)}{[1+(-2)^2]^2}(2) = \frac{8}{25}$ cm/sec

(7) Given

$y = \tan x$

$\frac{dx}{dt} = 2$ cm/sec

Find

$\frac{dy}{dt}$ when $x = -\frac{\pi}{3}$

$\frac{dy}{dt} = \sec^2(x) \cdot \frac{dx}{dt}$

$\frac{dy}{dt} = \left[\sec\left(-\frac{\pi}{3}\right)\right]^2 \cdot 2$

$2^2 \cdot 2$

$= 8$ cm/sec

(8) Given

$y = \sin x$

$\frac{dx}{dt} = 2$ cm/sec

Find

$\frac{dy}{dt}$ when $x = \frac{\pi}{6}$

$\frac{dy}{dt} = (\cos x)\left(\frac{dx}{dt}\right)$

$\frac{dy}{dt} = \cos\frac{\pi}{6} \cdot 2$

$= \frac{2\sqrt{3}}{2} = \sqrt{3}$ cm/sec

13. Given $(0,0)$ (x, x^2+1)

Find $\frac{dd}{dt}$

$d = \sqrt{(x-0)^2 + (x^2+1-0)^2}$

$d = \sqrt{x^2 + (x^2+1)^2}$

$\frac{dx}{dt} = 2 \text{ cm/sec}$

$$d = \left[x^2 + (x^2+1)^2\right]^{\frac{1}{2}} = \left(x^2 + x^4 + 2x^2 + 1\right)^{\frac{1}{2}} = \left(x^4 + 3x^2 + 1\right)^{\frac{1}{2}}$$

$$\frac{dd}{dt} = \frac{1}{2}\left(x^4+3x^2+1\right)^{-\frac{1}{2}}\left(4x^3+6x\right)\left(\frac{dx}{dt}\right)$$

$$\frac{dd}{dt} = \frac{4x^3+6x}{2\left(x^4+3x^2+1\right)^{1/2}}(2) = \boxed{\frac{4x^3+6x}{\left(x^4+3x^2+1\right)^{1/2}}}$$

15. Given

$A = \pi r^2$

$\frac{dr}{dt} = 3 \text{ cm/min}$

$\frac{dA}{dt} = 2\pi r \cdot \frac{dr}{dt}$

Find

a. $\frac{dA}{dt}$ when $r = 6$

b. $\frac{dA}{dt}$ when $r = 24$

(a) $\frac{dA}{dt} = 2\pi(6)(3)$

$= \boxed{36\pi \text{ cm}^2/\text{min}}$

(b) $\frac{dA}{dt} = 2\pi(24)(3)$

$= \boxed{144\pi \text{ cm}^2/\text{min}}$

16. $A = \pi r^2$

$\frac{dA}{dt} = 2\pi r \cdot \frac{dr}{dt}$

$\frac{dr}{dt}$ is constant.

$\frac{dA}{dt}$ is dependant on r, so it is not constant.

18. Given

$V = \frac{4}{3}\pi r^3$

$\frac{dr}{dt} = 2$ in/min

a) Find $\frac{dV}{dt}$ when $r=6$ and $r=24$

$\frac{dV}{dt} = 4\pi r^2 \left(\frac{dr}{dt}\right)$

$\frac{dV}{dt} = 4\pi(6)^2(2)$

$\boxed{\frac{dV}{dt} = 288\pi \text{ in}^3/\text{min}}$

$\frac{dV}{dt} = 4\pi(24)^2(2)$

$\boxed{= 4608 \text{ in}^3/\text{min}}$

b. It is dependant on r^2.

19. Given

$V = \frac{4}{3}\pi r^3$

$\frac{dV}{dt} = 800 \text{ cm}^3/\text{min}$

Find $\frac{dr}{dt}$ when (a) $r=30$ and (b) $r=60$

$\frac{dV}{dt} = 4\pi r^2 \left(\frac{dr}{dt}\right)$

a. $800 = 4\pi(30)^2 \cdot \frac{dr}{dt}$

$800 = 3600\pi \cdot \frac{dr}{dt}$

$\frac{dr}{dt} = \frac{800}{3600\pi} = \boxed{\frac{2}{9\pi} \text{ cm/min}}$

b. $800 = 4\pi(60)^2 \cdot \frac{dr}{dt}$

$800 = 14400\pi \cdot \frac{dr}{dt}$

$\frac{800}{14400\pi} = \frac{dr}{dt}$

$\boxed{\frac{dr}{dt} = \frac{1}{18\pi} \text{ cm/min}}$

20. Given
$V = x^3$
$\frac{dx}{dt} = 3\ cm/sec$

Find
$\frac{dV}{dt}$ when $x=1$ and $x=10$

$\frac{dV}{dt} = 3x^2\left(\frac{dx}{dt}\right)$

@ 1 sec
$\frac{dV}{dt} = 3(1)^2(3)$
$= 9\ cm^3/sec$

@ 10 sec
$\frac{dV}{dt} = 3(10)^2(3)$
$= 900\ cm^3/sec$

21. Given
$S = 6x^2$
$\frac{dx}{dt} = 3\ cm/sec$

Find
$\frac{dS}{dt}$ when $x=1$ and $x=10$

$\frac{dV}{dt} = 12x\left(\frac{dx}{dt}\right)$

@ 1 sec
$\frac{dV}{dt} = 12(1)(3)$
$= 36\ cm^2/sec$

@ 10 sec
$\frac{dV}{dt} = 12(10)(3)$
$= 360\ cm^2/sec$

22. Given
$V = \frac{1}{3}\pi r^2 h$
$\frac{dr}{dt} = 2\ in/min$
$h = 3r$

Find
$\frac{dV}{dt}$ when $r=6$ and $r=24$

$V = \frac{1}{3}\pi r^2(3r)$
$V = \pi r^3$
$\frac{dV}{dt} = 3\pi r^2\left(\frac{dr}{dt}\right)$

$\frac{dV}{dt} = 3\pi(6)^2(2)$
$= 216\pi\ in^3/min$

$\frac{dV}{dt} = 3\pi(24)^2(2)$
$= 3456\pi\ in^3/min$

(23.) Given

- $V = \frac{1}{3}\pi r^2 h$
- $\frac{dV}{dt} = 10\ ft^3/min$
- $d = 3h \rightarrow 2r = 3h$
  $r = \frac{3}{2}h$

$V = \frac{1}{3}\pi\left(\frac{3}{2}h\right)^2 h$

$V = \frac{1}{3}\pi\left(\frac{9}{4}h^2\right)\cdot h$

$V = \frac{3}{4}\pi h^3$

Find $\frac{dh}{dt}$ when $h=15$

$\rightarrow \frac{dV}{dt} = \frac{9}{4}\pi h^2 \cdot \left(\frac{dh}{dt}\right)$

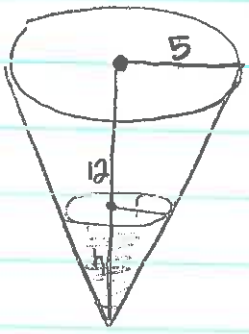
$10 = \frac{9}{4}\pi(15)^2 \cdot \frac{dh}{dt}$

$10 = \frac{2025\pi}{4} \cdot \frac{dh}{dt}$

$10\left(\frac{4}{2025\pi}\right) = \frac{dh}{dt}$

$\frac{8}{405\pi}\ ft/min = \frac{dh}{dt}$

(24.)



Given

$V = \frac{1}{3}\pi r^2 h$

$\frac{dV}{dt} = 10\ ft^3/min$

Find $\frac{dh}{dt}$ when $h=8$

$\frac{5}{r} = \frac{12}{h} \Rightarrow 5h = 12r$

$\frac{5}{12}h = r$

$V = \frac{1}{3}\pi\left(\frac{5}{12}h\right)^2 \cdot h$

$V = \frac{25\pi}{432} \cdot h^3$

$\frac{dV}{dt} = 3\left(\frac{25\pi}{432}\right)h^2 \cdot \frac{dh}{dt}$

$\frac{dV}{dt} = \frac{25\pi}{144} \cdot h^2 \cdot \frac{dh}{dt}$

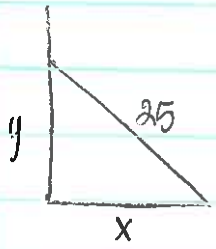
$10 = \frac{25\pi}{144}(8)^2 \cdot \frac{dh}{dt}$

$10 = \frac{1600\pi}{144} \cdot \frac{dh}{dt}$

$\frac{1440}{1600\pi} = \frac{dh}{dt}$

$\frac{9}{10\pi}\ ft/min$

27.



Given

$x^2+y^2=625$

$\frac{dx}{dt}=2$ ft/sec

Find

$\frac{dy}{dt}$ when $x=7, x=15, x=24$

$\downarrow$ $y=24$ $\quad$ $\downarrow$ $y=20$ $\quad$ $\downarrow$ $y=7$

a.

$$2x\left(\frac{dx}{dt}\right)+2y\left(\frac{dy}{dt}\right)=0$$

$$2y\left(\frac{dy}{dt}\right)=-2x\left(\frac{dx}{dt}\right)$$

$$\frac{dy}{dt}=\frac{-2x}{2y}\left(\frac{dx}{dt}\right)$$

$$\frac{dy}{dt}=\frac{-x}{y}\left(\frac{dx}{dt}\right)$$

when $x=7$

$\frac{dy}{dt}=\frac{-7}{24}(2)=\boxed{\frac{-7}{12}\text{ ft/sec}}$

when $x=15$

$\frac{dy}{dt}=\frac{-15}{20}(2)=\frac{-15}{10}=\boxed{\frac{-3}{2}\text{ ft/sec}}$

when $x=24$

$\frac{dy}{dt}=\frac{-24}{7}(2)=\boxed{\frac{-48}{7}\text{ ft/sec}}$

b.

Given

$A=\frac{1}{2}xy$

$$\frac{dA}{dt}=\frac{1}{2}x\left(\frac{dy}{dt}\right)+\frac{1}{2}y\left(\frac{dx}{dt}\right)$$

(from part a)

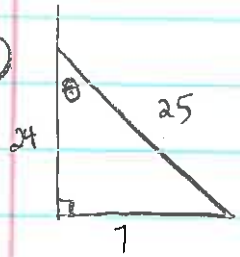
$$\frac{dA}{dt}=\frac{1}{2}(7)\left(\frac{-7}{12}\right)+\frac{1}{2}(24)(2)$$

$\frac{-49}{24}+24$

$=527/24\text{ ft}^2/\text{sec}$

$\approx 21.96\text{ ft}^2/\text{sec}$

c.



Given

$\sin\theta=\frac{x}{25}$

when $x=7$,

$\theta=\sin^{-1}\left(\frac{7}{25}\right)$

$$(\cos\theta)\cdot\frac{d\theta}{dt}=\frac{1}{25}\cdot\frac{dx}{dt}$$

$$\frac{d\theta}{dt}=\frac{1}{25\cos\theta}\cdot\frac{dx}{dt}$$

$$\frac{d\theta}{dt}=\frac{2}{25\cos\left(\sin^{-1}\left(\frac{7}{25}\right)\right)}$$

$=\frac{2}{24}=\boxed{\frac{1}{12}\text{ rad/sec}}$

OR

$\frac{25}{24}\cdot\frac{24}{25}\cdot\frac{d\theta}{dt}=\frac{1}{25}\cdot(2)\frac{25}{24}$

$\frac{d\theta}{dt}=\frac{1}{12}\text{ rad/sec}$

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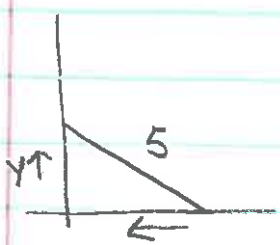

Given: $x^2+y^2=25$

$\frac{dy}{dt} = 0.15 \text{ m/sec}$

Find:

$\frac{dx}{dt}$ when $x=2.5$

$\downarrow$

$y=\sqrt{18.75}$

$$2x\left(\frac{dx}{dt}\right) + 2y\left(\frac{dy}{dt}\right) = 0$$

$$2(2.5)\left(\frac{dx}{dt}\right) + 2\sqrt{18.75}\,(0.15) = 0$$

$$\frac{dx}{dt} = \frac{-0.3\sqrt{18.75}}{5} \approx -0.26 \text{ m/sec}$$