Chapter 1 Review
Calculus

Name: Key

Answer each of the following and show all work.

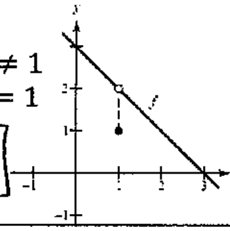
1. $\lim\limits_{x\to 2} 3x^2+5 = \boxed{17}$

$3(2)^2+5$

2. Use the graph to find $\lim_{x\to 1} f(x)$ if

$f(x) = \begin{cases} 3-x, & x \neq 1 \\ 1, & x = 1 \end{cases}$

$\lim\limits_{x\to 1} f(x) = \boxed{2}$



$\frac{0}{0}$ 3. $\lim\limits_{x\to -1} \dfrac{x+1}{x^3-x} = \lim\limits_{x\to -1} \dfrac{(x+1)}{x(x+1)(x-1)}$

$= \lim\limits_{x\to -1} \dfrac{1}{x(x-1)} = \dfrac{1}{-1(-1-1)} = \boxed{\dfrac{1}{2}}$

Find the limit algebraically. $\frac{0}{0}$

3. $\lim\limits_{x\to 4} \dfrac{x-4}{x^2-3x-4} = \dfrac{1}{5}$

$\lim\limits_{x\to 4} \dfrac{x-4}{(x-4)(x+1)} = \lim\limits_{x\to 4} \dfrac{1}{x+1}$

$= \dfrac{1}{5}$

Find the limit algebraically.

4. $\lim\limits_{x\to 3} \sqrt{x^2-4} = \sqrt{5}$

$\sqrt{9-4}$

5. If $\lim_{x\to c} f(x) = -\frac{1}{2}$ and $\lim_{x\to c} g(x) = \frac{3}{2}$, find $\lim_{x\to c} \frac{f(x)}{g(x)}$.

$\dfrac{-\frac{1}{2}}{\frac{3}{2}} = \dfrac{-1}{3}$ $\quad \lim\limits_{x\to c} \dfrac{f(x)}{g(x)} = \dfrac{-1}{3}$

Find the limit algebraically.

6. $\lim\limits_{x\to 2} \dfrac{x-2}{x^2-4} = \dfrac{1}{4}$

$\lim\limits_{x\to 2} \dfrac{1}{x+2} = \dfrac{1}{4}$

7. $\lim\limits_{x\to 3^+} \sqrt{2x-5} = 1$

$\sqrt{6-5} = \sqrt{1} = 1$

8. Fill in the table and approximate the limit.

$\lim\limits_{x\to 3} \dfrac{x-3}{|x-3|} = \text{DNE}$

x	2.9	2.99	2.999	3	3.001	3.01	3.1
f(x)	-1	-1	-1		1	1	1

9. $\lim\limits_{x\to 2} \sec\dfrac{\pi x}{3} = \boxed{-2}$

$\sec\dfrac{2\pi}{3} = \dfrac{1}{\cos\frac{2\pi}{3}}$

$= \dfrac{1}{-\frac{1}{2}} = -2$

Determine this limit graphically.

10. $\lim\limits_{x\to 0} \dfrac{x}{\tan x} = 1$

Find this limit by rationalizing the numerator.

11. $\lim\limits_{x\to 0} \dfrac{\sqrt{x+4}-2}{x} = \boxed{\dfrac{1}{4}}$

$\lim\limits_{x\to 0} \dfrac{\sqrt{x+4}-2}{x} \cdot \dfrac{\sqrt{x+4}+2}{\sqrt{x+4}+2} = \lim\limits_{x\to 0} \dfrac{x}{x(\sqrt{x+4}+2)}$

$= \lim\limits_{x\to 0} \dfrac{1}{\sqrt{x+4}+2} = \dfrac{1}{4}$

12. $\lim\limits_{x\to 2^-} \dfrac{1}{x-2} = \dfrac{1}{0^-} = -\infty$

13. $\lim\limits_{x\to 2^-} \dfrac{1}{(x-2)^2} = \dfrac{1}{0^+} = \infty$

$\lim\limits_{x\to 2^+} \dfrac{1}{(x-2)^2} = \dfrac{1}{0^+} = \infty$

$\lim\limits_{x\to 2} \dfrac{1}{(x-2)^2} = \infty$

14. Discuss the continuity of $f(x) = \frac{x^2-2x-3}{x-2}$.

$\dfrac{(x-3)(x+1)}{(x-2)} = f(x)$

f(x) has a nonremovable discontinuity at $x=2$

15.

a). $\lim\limits_{x\to -1^+} f(x) = -\frac{3}{2}$

b). $\lim\limits_{x\to -1^-} f(x) = -3$

c). $\lim\limits_{x\to -1} f(x) = \text{DNE}$

d). $\lim\limits_{x\to 5} f(x) = 5$

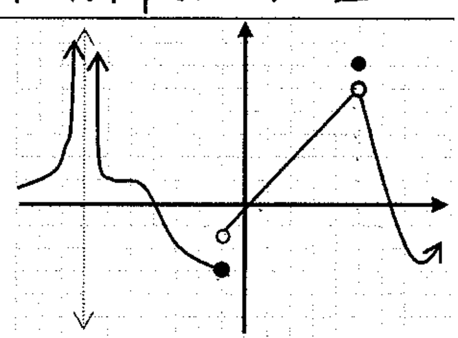
e). $\lim\limits_{x\to -7^-} f(x) = \infty$

f). $\lim\limits_{x\to -7} f(x) = \infty$

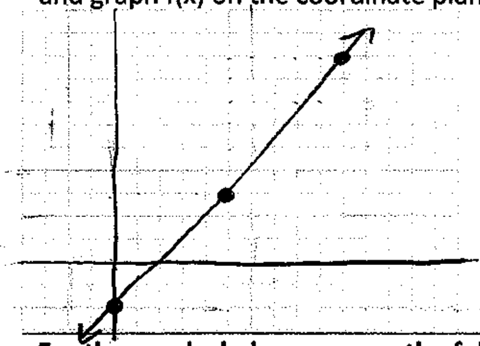
g). $f(-1) = -3$

h). $f(5) = 6$

f). $f(-7) = \text{undefined}$



18. Determine the value of *c* so that f(x) is continuous on the entire real line when $f(x) = \begin{cases} x-2, & x \le 5 \\ cx-3, & x > 5 \end{cases}$ and graph f(x) on the coordinate plane below.

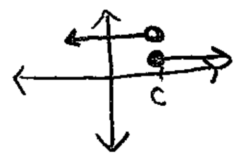
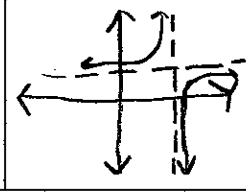
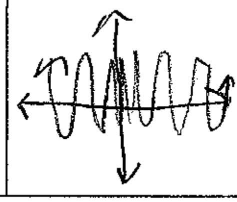


$3 = c(5) - 3$

$6 = 5c$

$c = \frac{6}{5}$

19. What are the conditions under which limits fail to exist? Draw and example of each case and explain why the limit DNE.

$\lim_{x\to c^-} f(x) \neq \lim_{x\to c^+} f(x)$	unbounded behavior	osscilating behavior
		

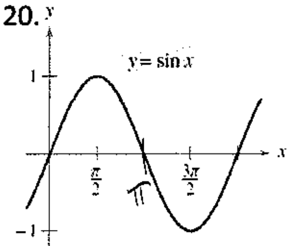
For the graphs below, answer the following:

a) Give the right sided limit
b) Give the left sided limit
c) Determine the overall limit
d) Determine if the function is continuous at *c*. If discontinuous, then is it removable or non-removable?
e) If it is not continuous at *c*, tell which of the 3 condition it fails.

Conditions for continuity at a point:

1. *f(c)* is defined
2. $\lim_{x\to c} f(x)$ exists
3. $\lim_{x\to c} f(x) = f(c)$

20.



a) $\lim_{x\to\pi^+} f(x) = 0$

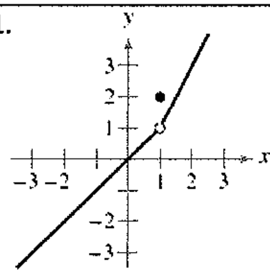
b) $\lim_{x\to\pi^-} f(x) = 0$

c) $\lim_{x\to\pi} f(x) = 0$

d) continuous

e)

21.



a) $\lim_{x\to 1^+} f(x) = 1$

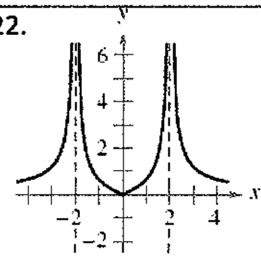
b) $\lim_{x\to 1^-} f(x) = 1$

c) $\lim_{x\to 1} f(x) = 1$

d) discontinuous removable

e) $\lim_{x\to c} f(x) \neq f(c)$

22.



a) $\lim_{x\to 2^+} f(x) = \infty$

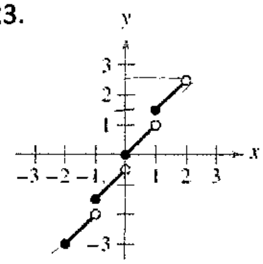
b) $\lim_{x\to 2^-} f(x) = \infty$

c) $\lim_{x\to 2} f(x) = \infty$

d) discontinuous nonremovable

e) $\lim_{x\to c} f(x)$ dne

23.



a) $\lim_{x\to -1^+} f(x) = \frac{3}{2}$

b) $\lim_{x\to -1^-} f(x) = -2$

c) $\lim_{x\to -1} f(x) =$ DNE

d) discontinuous nonremovable

e) $\lim_{x\to c} f(x)$ DNE

Find all the vertical asymptote(s) of $f(x) = \frac{2x-3}{2x^2+x-3}$. Show algebraic support for your answer(s).

$f(x) = \frac{2x-3}{(2x+3)(x-1)}$

VA at $x = \frac{-3}{2}$ and $x = 1$

$2x+3=0 \quad x-1=0$

$x = \frac{-3}{2} \quad x = 1$