

Chapter 1

$$\textcircled{1} \lim_{x \rightarrow 5} \frac{x^2 - 25}{x - 5} = \lim_{x \rightarrow 5} \frac{(x-5)(x+5)}{(x-5)} = \lim_{x \rightarrow 5} x+5 = \boxed{10}$$

$$\textcircled{2} f(x) = \frac{3x^2 - x - 2}{x-1} = \frac{(3x+2)(x-1)}{x-1} = 3x+2 \quad \boxed{\text{Removable Disc.}} \\ \textcircled{x=1}$$

$$\textcircled{3} \lim_{x \rightarrow 2^-} \frac{1}{\sqrt{x^2 - 4}} = \boxed{-\infty} \\ \text{(DNE)}$$

$$\textcircled{4} \text{ a. } \lim_{x \rightarrow 0} f(x) \quad \boxed{\text{DNE}} \quad \text{ b. } f(0) = \boxed{5} \quad \text{ c. } \lim_{x \rightarrow -4} f(x) = \boxed{2}$$

$$\text{ d. } f(-4) = \boxed{0} \quad \text{ e. } \lim_{x \rightarrow -2} f(x) \quad \boxed{\text{DNE}} \\ \text{-}\infty$$

$$\textcircled{5} \lim_{x \rightarrow 3} \sqrt{x^2 - 4} = \boxed{\sqrt{5}} \quad \text{ (b.) } \lim_{x \rightarrow 2^-} \frac{1}{x-2} = \boxed{-\infty} \\ \text{(DNE)}$$

$\textcircled{7}$ A discontinuity at c is removable if the function can be made continuous by defining or redefining $f(c)$; for example: A hole.

A nonremovable discontinuity is one in which a function cannot be redefined to make it continuous, for example: VA or jump.

Chapter 2

① $f(x) = x^2 + 5x - 13$

$$\lim_{h \rightarrow 0} \frac{[(x+h)^2 + 5(x+h) - 13] - (x^2 + 5x - 13)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{x^2 + 2xh + h^2 + 5x + 5h - 13 - x^2 - 5x - 13}{h} = \lim_{h \rightarrow 0} \frac{2xh + h^2 + 5h}{h}$$

$$= \lim_{h \rightarrow 0} 2x + h + 5 = \boxed{2x + 5}$$

✓ ② $f(x) = 6\sqrt{x} - \frac{3}{2x^3} + \cos x$

$$f(x) = 6x^{1/2} - \frac{3}{2}x^{-3} + \cos x$$

$$f'(x) = 3x^{-1/2} + \frac{9}{2}x^{-4} - \sin x$$

$$f'(x) = \frac{3}{\sqrt{x}} + \frac{9}{2x^4} - \sin x$$

③ $f(x) = (2x^2 - 7)^5$

$$f'(x) = 5(2x^2 - 7)^4(4x)$$

$$f'(x) = 20x(2x^2 - 7)^4$$

✓ ④ $f(x) = 3x^2 \sec x$

$$f'(x) = 3x^2(\sec x \tan x) + 6x \sec x$$

✓ ⑤ $f(x) = \frac{\tan x}{x^2}$

$$f'(x) = \frac{x^2 \sec^2 x - 2x \tan x}{x^4}$$

$$f'(x) = \frac{x \sec^2 x - 2 \tan x}{x^3}$$

⑥ $f(x) = 2x^2 + \sin^2(2x)$

$$f'(x) = 4x + 2(\sin(2x))(\cos(2x))(2)$$

$$f'(x) = 4x + 4 \sin 2x \cos 2x$$

$$\checkmark (7) \quad x^2 + 3xy + y^3 = 10$$

$$2x + 3x\left(\frac{dy}{dx}\right) + 3y + 3y^2\left(\frac{dy}{dx}\right) = 0$$

$$(3x + 3y^2)\frac{dy}{dx} = -2x - 3y$$

$$\boxed{\frac{dy}{dx} = \frac{-2x - 3y}{3x + 3y^2}}$$

$$\checkmark (8) \quad f(x) = \frac{2}{3}x^2 - \frac{1}{6}x \quad \left(-1, \frac{5}{6}\right)$$

$$f'(x) = \frac{4}{3}x - \frac{1}{6}$$

$$f'(-1) = -\frac{4}{3} - \frac{1}{6} = -\frac{3}{2}$$

$$\boxed{y - \frac{5}{6} = -\frac{3}{2}(x+1)}$$

$$\checkmark (9) \quad v_0 = 0$$

$$a(t) = -32 \text{ ft/sec}$$

$$v(t) = -32t$$

$$s(t) = -16t^2 + h$$

$$0 = -16(9.2)^2 + h$$

$$\boxed{1354.24 \text{ ft} = h}$$

$$(10) \quad SA = 6x^2$$

$$\frac{ds}{dt} = 12x \left(\frac{dx}{dt}\right)$$

$$\frac{ds}{dt} = 12(4.5)(5) = \boxed{270 \text{ cm/sec}}$$

$$\checkmark (11) \quad f(x) = \frac{x+1}{x-1} \quad \left(\frac{1}{2}, -3\right)$$

$$f'(x) = \frac{(x-1) - (x+1)}{(x-1)^2} = \frac{-2}{(x-1)^2}$$

$$f'\left(\frac{1}{2}\right) = 8$$

$$\boxed{y + 3 = 8\left(x - \frac{1}{2}\right)}$$

$$\checkmark (12) \quad f(x) = \sqrt[3]{1-x^3}$$

$$f'(x) = \frac{1}{3}(1-x^3)^{-2/3}(-3x^2)$$

$$\boxed{f'(x) = \frac{-x^2}{(1-x^3)^{2/3}}}$$

$$\checkmark (13) \quad v(t) = \frac{90t}{4t+10}$$

$$v'(t) = \frac{(4t+10)(90) - 90t(4)}{(4t+10)^2}$$

$$\checkmark (14) \quad y = 1 - \cos(2x) + 2\cos^2 x$$

$$\boxed{\frac{dy}{dx} = 2\sin(2x) - 4\cos x \sin x}$$

$$v'(5) = \frac{30(90) - 1800}{(30)^2}$$

$$\checkmark (15) \quad x^2 + y^2 = 20 \quad (2, 4)$$

$$2x + 2y\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = -\frac{x}{y} \quad @ (2, 4) = -\frac{1}{2}$$

$$\boxed{v'(5) = 1 \text{ ft/sec}^2}$$

$$\boxed{y - 4 = -\frac{1}{2}(x - 2)}$$

Chapter 3

① $f(x) = x^3 - 12x$

$f'(x) = 3x^2 - 12$

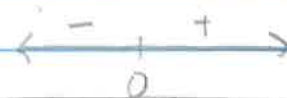
$f''(x) = 6x$

$0 = 3x^2 - 12$

$0 = 6x$

$x = \pm 2$

$x = 0$



Rel Max $(-2, 16)$ Rel Min $(2, -16)$

CU: $(0, \infty)$ CD: $(-\infty, 0)$

Inc: $(-\infty, -2) \cup (2, \infty)$ Dec: $(-2, 2)$

POI $(0, 0)$

② $f(x) = 4x + 8 \cos x$

$f'(x) = 4 - 8 \sin x$

$f''(x) = -8 \cos x$

$f(\frac{\pi}{6}) = \frac{2\pi}{3} + 8(\frac{\sqrt{3}}{2})$

$0 = 4 - 8 \sin x$

$-8 \cos x = 0$

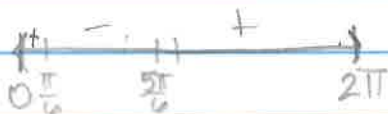
$f(\frac{5\pi}{6}) = \frac{5\pi}{3} - 8(\frac{\sqrt{3}}{2})$

$\sin x = \frac{1}{2}$

$\cos x = 0$

$x = \frac{\pi}{6}, \frac{5\pi}{6}$

$x = \frac{\pi}{2}, \frac{3\pi}{2}$



Rel Max $(\frac{\pi}{6}, \frac{2\pi}{3} + 4\sqrt{3})$ Rel Min $(\frac{5\pi}{6}, \frac{10\pi}{3} - 4\sqrt{3})$

CU: $(\frac{\pi}{2}, \frac{3\pi}{2})$ CD: $(0, \frac{\pi}{2}) \cup (\frac{3\pi}{2}, 2\pi)$

Inc $(0, \frac{\pi}{6}) \cup (\frac{5\pi}{6}, 2\pi)$

POI $(\frac{\pi}{2}, 2\pi) + (\frac{3\pi}{2}, 6\pi)$

③ $f'(x) = 2x + 2$

x	y = f(x)
-2	0
-1	-1
4	24

Abs Min $(-1, -1)$

$x = -1$

$-1 - 2 = -1$

Abs Max $(4, 24)$

$f(x) = x^2 + 2x$

$4 = 24$

④ $\lim_{x \rightarrow \infty} \frac{-2x^2 - 6x + 1}{3x^2 - 2} = -\infty$

⑤ $\lim_{x \rightarrow \infty} \frac{x^2 + x^3}{x^2 - 3} = \infty$

⑥ $\lim_{x \rightarrow \infty} \frac{-4x^3 + x + 1}{x^3 - 2} = 0$

✓ ⑦ $f(x) = 2x^2 - 3x + 1$

$(0, 1) (4, 21)$

$m = \frac{20}{4} = 5$

$f'(x) = 4x - 3$

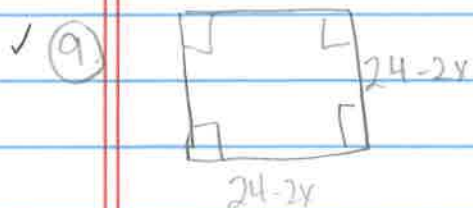
$4x - 3 = 5$

$4x = 8$

$x = 2$

$c = 2$

8.) $f(x)$ is defined at $x=3$, increasing, and concave down



$$V = x(24-2x)(24-2x)$$

$$V = x(576 - 48x - 4x^2)$$

$$V = 576x - 48x^2 - 4x^3$$

$$\frac{dV}{dx} = 576 - 96x - 12x^2$$

$$x = \frac{96 \pm \sqrt{9216 - 4(-12)(576)}}{2(-12)}$$

$$x = \frac{96 \pm 192}{-24} = 4 \text{ or } -12$$

$$x = 4 \quad \text{Max } V = \boxed{1024 \text{ in}^3}$$

10.) $f(x) = 3x^2 + 2x + 1$

$$f'(x) = 6x + 2$$

$$\boxed{x = -\frac{1}{3}}$$

Chapter 4

① $\int (5-x) dx = 5x - \frac{1}{2}x^2 + C$

③ $\int (x^{3/4} + 1) dx$
 $= \frac{4}{7}x^{7/4} + x + C$

④ $\int (x^2 + 3x^3) dx$
 $= \frac{1}{3}x^3 + \frac{3}{4}x^4 + C$

⑥ $\int -2 \cos^4 x (\sin x) dx$
 $u = \cos x \quad -2 \int u^4 du$
 $du = -\sin x dx \quad = -2 \left[\frac{1}{5} u^5 \right] + C$
 $= -\frac{2}{5} \cos^5 x + C$

⑦ $\int 5x (2-x^2)^8 dx$
 $u = 2-x^2 \quad = -\frac{5}{2} \int u^8 du$
 $du = -2x dx \quad = -\frac{5}{2} \left[\frac{1}{9} u^9 \right] + C$
 $= -\frac{5}{18} (2-x^2)^9 + C$

⑨ $\int (x^{-2} + 2 - 3x^{-4}) dx$
 $= -x^{-1} + 2x + \frac{3}{3}x^{-3} + C$
 $= -\frac{1}{x} + 2x + \frac{1}{x^3} + C$

⑩ $\int_{-2}^4 (3x-5) dx = -\left[\frac{3}{2}x^2 - 5x \right]_{-2}^4$
 $= 4 - 16 = -12$

x ⑪ $\int_1^3 (t^{-2} - t^{-4}) dt = \left[-t^{-1} + \frac{1}{3}t^{-3} \right]_1^3$
 $\left[-\frac{1}{t} + \frac{1}{3t^3} \right]_1^3 = \frac{-26}{81} + \frac{2}{3} = \frac{28}{81}$
 ≈ 0.3457

x ⑫ $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \sec^2 \theta d\theta$
 $= [\tan \theta]_{\frac{\pi}{4}}^{\frac{\pi}{2}}$
 $= \tan \frac{\pi}{2} - \tan \frac{\pi}{4}$
 $0 - 1 = -1$

② $\frac{1}{9} \int x^2 \sqrt{1-3x^3} dx$ $u = 1-3x^3$
 $du = -9x^2 dx$
 $= -\frac{1}{9} \int u^{1/2} du$
 $= -\frac{1}{9} \left[\frac{2}{3} u^{3/2} \right] + C$
 $= -\frac{2}{27} (1-3x^3)^{3/2} + C$

⑤ $\frac{1}{2} \int 2(2x-3)^3 dx$ $u = 2x-3$
 $du = 2 dx$
 $= \frac{1}{2} \int u^3 du$
 $= \frac{1}{2} \left[\frac{1}{4} u^4 \right] + C$
 $= \frac{1}{8} (2x-3)^4 + C$

$$\begin{aligned}
 (13) \quad & \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} (\cos\theta + 2\sin\theta) d\theta \\
 & = [\sin\theta - 2\cos\theta]_{\frac{\pi}{4}}^{\frac{\pi}{2}} \\
 & (0+2) - \left(\frac{\sqrt{2}}{2} - 2\left(\frac{\sqrt{2}}{2}\right)\right) \\
 & = 2 - \frac{\sqrt{2}}{2} + \sqrt{2} \\
 & = 2 + \frac{\sqrt{2}}{2} = \boxed{\frac{4+\sqrt{2}}{2}}
 \end{aligned}$$

$$\begin{aligned}
 (14) \quad & f(x) = x^2 - 2x \quad [0, 3] \\
 & \frac{1}{3} \int_0^3 (x^2 - 2x) dx \\
 & \frac{1}{3} \left[\frac{1}{3}x^3 - x^2 \right]_0^3 \\
 & \frac{1}{3} [0 - 0] = \boxed{0}
 \end{aligned}$$

$$\begin{aligned}
 (15) \quad & f(x) = \sin x \quad [0, \pi] \\
 & \frac{1}{\pi} \int_0^{\pi} \sin x dx \\
 & \frac{1}{\pi} [-\cos x]_0^{\pi} \\
 & \frac{1}{\pi} (1 + 1) = \boxed{\frac{2}{\pi}}
 \end{aligned}$$

$$\begin{aligned}
 (16) \quad & \int_{-0.5}^1 \sqrt{1+x^3} dx = \frac{1}{2} \left(\frac{1}{2}\right) [f(-0.5) + 2f(0) + 2f(0.5) + f(1)] \\
 & = \frac{1}{4} [6.471] = \boxed{1.618}
 \end{aligned}$$