Chapter 1 Review

1. Complete the table and use the result to estimate the limit.

$$\lim_{x\to 5}\frac{\frac{1}{x+3}-\frac{1}{8}}{x-5}$$

x	4.9	4.99	4.999	5.001	5.01	5.1
$f(x)$						

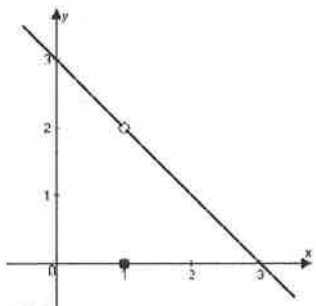
A) 0.114375 B) 0.094375 C) –0.145625 D) –0.015625 E) –0.125625

2. Let

$$f(x)=\begin{cases}3-x, & x\neq 1\\ 0 & x=1\end{cases}.$$

Determine the following limit. (Hint: Use the graph of the function.)

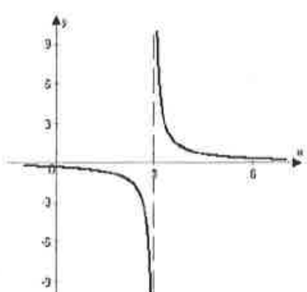
$$\lim_{x\to 1} f(x)$$



A) 2 B) 0 C) 3 D) 4 E) Does not exist

3. Determine the following limit. (Hint: Use the graph of the function.)

$$\lim_{x\to 3}\frac{1}{x-3}$$



A) 0 B) Does not exist C) 3 D) –3 E) –6

4. Find the limit:

$$\lim_{x\to 5}\cos\left(\frac{\pi x}{6}\right)$$

$\cos\frac{5\pi}{6}$ $\frac{\sqrt{3}}{2}$

A) $\frac{3^{-1/2}}{4}$ B) $\frac{3^{1/2}}{2}$ C) $-\frac{3^{1/2}}{2}$ D) 0 E) $-\frac{3^{-1/2}}{4}$

5. Find the limit (if it exists):

$$\lim_{x\to -8}\frac{-x-8}{x^2-64}$$

$$\frac{-(x+8)}{(x-8)(x+8)}=\frac{-1}{x-8}=\frac{-1}{-16}=\frac{1}{16}$$

A) 32 B) $-\frac{1}{16}$ C) $\frac{1}{16}$ D) –8 E) $\frac{1}{32}$

6. Find the x-values (if any) at which the function $f(x)=\dfrac{x-5}{x^2-6x+5}$ is not continuous. hole

Which of the discontinuitites are removable?

A) No points of discontinuity.
B) $x=5$ (Not removable), $x=1$ (Removable)
C) $x=5$ (Removable), $x=1$ (Not removable)
D) No points of continuity.
E) $x=5$ (Not removable), $x=1$ (Not removable)

$$\frac{\cancel{x-5}}{(\cancel{x-5})(x-1)} = \frac{1}{x-1}$$

$x=1$ asymptote.

7. Find the vertical asymptotes (if any) of the function $f(x)=\dfrac{x^2-25}{x^2-15x+50}$.

A) $x=-10$ B) $x=10$ C) $x=5$ D) $x=-5$ E) $x=-50$

hole

$$\frac{(\cancel{x-5})(x+5)}{(\cancel{x-5})(x-10)} = \frac{x+5}{x-10}$$

$x=10$

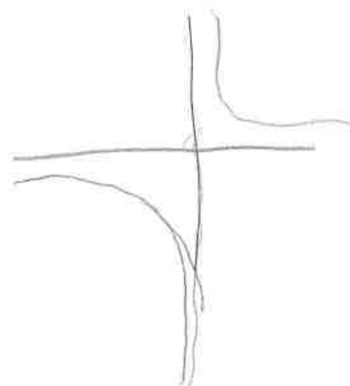
8. Find the limit:

$$\lim_{x\to 0^-}\left(x^4-\frac{1}{x}\right)$$

A) 0 B) $-\infty$ C) 1 D) -1 E) ∞

$$\lim_{x\to 0^-} x^4 - \lim_{x\to 0^-} \frac{1}{x}$$

$$\lim_{x\to c} x^n = c^n$$



$$0^4 - -\infty$$

$$0 + \infty = \infty$$

Chapter 2 Review

1. Find the derivative of the following function using the limiting process.

$$\lim_{x\to h} \frac{-4(x+h)^2 + 6(x+5) - (-4x^2+6x)}{h}$$

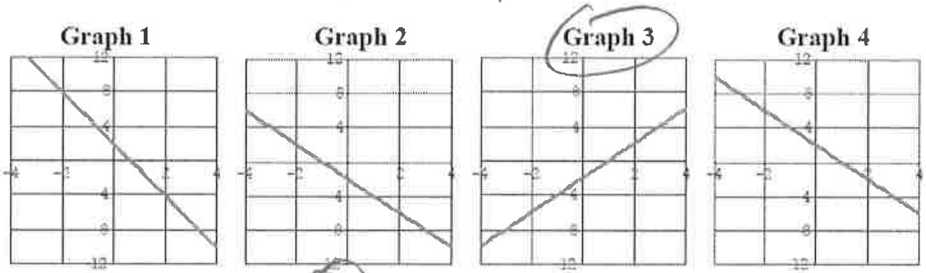
$f(x) = -4x^2 + 6x$

A) -4 B) $-8x+6$ C) $-8x-6$ D) $-8x$ E) None of the above

2. Identify the graph which has the following characteristics.

$f(0) = -2$

$f'(x) = 2,\ -\infty < x < \infty$ → $f(x) = 2x - 2$



A) Graph 2 B) Graph 1 C) Graph 3 D) Graph 4 E) None of the above

3. Find the derivative of the function.

$f(x) = \frac{1}{x^4} = \quad x^{-4} \quad f'(x) = -4x^{-5} = -\frac{4}{x^5}$

A) $f'(x) = -\frac{3}{x^5}$

B) $f'(x) = -\frac{4}{x^3}$

C) $f'(x) = -\frac{4}{x^5}$

D) $f'(x) = -\frac{5}{x^5}$

E) ~~None of the above~~

4. Find the derivative of the function.

$f'(x) = -16x + 4\sin x$

$f(x) = -8x^2 - 4\cos(x)$

A) $f'(x) = -8x + 4\sin(x)$

B) $f'(x) = -16x - 4\sin(x)$

C) $f'(x) = -16x + 4\sin(x)$

D) $f'(x) = -16x - 4\cos(x)$

E) None of the above

5. Find the slope of the graph of the function at the given value.

$f(x) = (5x-6)^2$ when $x = 5$

A) $f'(5) = 38$

B) $f'(5) = 190$

C) $f'(5) = 13718$

D) $f'(5) = 95$

E) $f'(5) = 310$

$f'(x) = 2(5x-6)^1 \cdot 5$

$= 10(5x-6)$

$= 50x - 60$

$f'(5) = 50(5) - 60$

$= 190$

6. Find the slope of the graph of the function at the given value.

$f(x) = 5x^2 + 9x - \frac{7}{x^2}$ when $x = -4$

A) $f'(-4) = -\frac{999}{32}$

B) $f'(-4) = \frac{999}{32}$

C) $f'(-4) = -\frac{999}{8}$

D) $f'(-4) = \frac{999}{8}$

E) $f'(-4) = -\frac{999}{2}$

-31.21875

$f'(x) = 10x + 9 + \frac{14}{x^3}$

$f'(4) =$

7. Find the slope of the graph of the function at the given value.

$f(x) = x(5x^5 + 9)$ when $x = 4$ $= 5x^6 + 9x$ $f'(x) = 30x^5 + 9$

A) $f'(4) = 6409$
B) $f'(4) = 30720$
C) $f'(4) = 30729$
D) $f'(4) = 25609$
E) $f'(4) = 30756$

$f(4) = 30729$

8. Determine the point(s), (if any), at which the graph of the function has a horizontal tangent.

$y(x) = x^4 - 32x + 3$

A) 0
B) 0 and 2
C) 0 and –2
D) 2
E) There are no points at which the graph has a horizontal tangent.

$y' = 4x^3 - 32$
$0 = 4x^3 - 32$
$4(x^3 - 8) = 0$
$\sqrt[3]{x^3} = \sqrt[3]{8}$
$x = 2$

9. Use the quotient rule to differentiate the following function and evaluate $H'(2)$.

$H(v) = \dfrac{4v}{v^4 + 3}$

A) $H'(2) = \dfrac{180}{361}$
B) $H'(2) = -\dfrac{180}{19}$
C) $H'(2) = \dfrac{180}{19}$
D) $H'(2) = -\dfrac{180}{361}$
E) $H'(2) = -\dfrac{180}{6859}$

$\dfrac{(v^4+3)4 - 4v(4v^3)}{(v^4+3)^2}$

$\dfrac{4v^4 + 12 - 16v^4}{(v^4+3)^2} = \dfrac{-180}{361}$

10. Find the derivative of the function.

$f(t) = 5t \sin t + 3\cos t$

$5t \cdot \cos t + 5\sin t - 3\sin t$

A) $f'(t) = -5t\sin t + 2\cos t$
B) $f'(t) = 5t\sin t - 5\cos t$
C) $f'(t) = 5t\sin t - 2\cos t$
D) $f'(t) = 5t\cos t + 2\sin t$
E) $f'(t) = 5t\cos t + 3\sin t$

11. The length of a rectangle is $3t + 3$ and its height is t^6, where t is time in seconds and the dimensions are in inches. Find the rate of change of area, A, with respect to time.

A) $\dfrac{dA}{dt} = t^6(18 + 21t)$ square inches/second
B) $\dfrac{dA}{dt} = t^5(18 + 21t)$ square inches/second
~~C)~~ $\dfrac{dA}{dt} = t^5(18 + 18t)$ inches/second
~~D)~~ $\dfrac{dA}{dt} = t^5(18 + 21t)$ inches/second
E) $\dfrac{dA}{dt} = t^5(21t + 3)$ square inches/second

$A = l \cdot w$
$A = (3t + 3)t^6$
$A = 3t^7 + 3t^6$
$\dfrac{dA}{dt} = 21t^6 + 18t^5$
$= t^5(21t + 18)$

12. Find the second derivative of the function.

$\frac{2}{5} - \frac{5}{5} = \frac{-3}{5}$

$f'(x) = \frac{2}{5} 7 x^{\frac{-3}{5}}$

$f(x) = 7x^{\frac{2}{5}}$

A) $f''(x) = \frac{-42}{25} x^{\frac{3}{5}}$

B) $f''(x) = \frac{2}{25} x^{\frac{-8}{5}}$

C) $f''(x) = \frac{175}{25} x^{\frac{-8}{5}}$

D) $f''(x) = \frac{-42}{25} x^{\frac{-8}{5}}$

E) None of the above

$f'(x) = 14 x^{\frac{-3}{5}}$

$f''(x) = -\frac{42}{5} x^{-8/5}$

13. Find the derivative of the function.

$y = \cos\left(6x^4 - 4\right)$

$-\sin\left(6x^4 - 4\right) \cdot 24x^3$

A) $y' = -24\sin\left(6x^4 - 4\right)$

B) $y' = -24x^3 \sin\left(6x^4 - 4\right)$

C) $y' = 24\sin\left(6x^4 - 4\right)$

D) $y' = 24x^4 \cos\left(6x^4 - 4\right)$

E) $y' = -6\sin\left(6x^4 - 4\right)$

14. Find *dy/dx* by implicit differentiation.

$2x + 9 + 9x\frac{dy}{dx} + 9y - 2y\frac{dy}{dx} = 0$

$x^2 + 9x + 9xy - y^2 = 16$

A) $\frac{dy}{dx} = \frac{x + 9 + 9y}{y - 9x}$

B) $\frac{dy}{dx} = \frac{2x + 9 + 9y}{2x - 9y}$

C) $\frac{dy}{dx} = \frac{2x - 9 + 9y}{2y - 9x}$

D) $\frac{dy}{dx} = \frac{2x + 9 - 9y}{2y - 9x}$

E) $\frac{dy}{dx} = \frac{2x + 9 + 9y}{2y - 9x}$

$(9x - 2y)\frac{dy}{dx} = -2x - 9 - 9y$

$\frac{dy}{dx} = \frac{-2x - 9 - 9y}{9x - 2y}$

15. Find an equation of the tangent line to the graph of the function given below at the given point.

$\cancel{f'(x)} = 14x - 2x\frac{dy}{dx} - 2y + 12y\frac{dy}{dx} = 0$

$7x^2 - 2xy + 6y^2 - 60 = 0, \quad (-2, 2)$

$(-2x + 12y)\frac{dy}{dx} = -14x + 2y$

(The coefficients below are given to two decimal places.)

A) $y = -1.14x + 4.29$

B) $y = 1.14x + 10.29$

C) $y = 1.14x + 4.29$

D) $y = 4.14x - 10.29$

E) $y = 4.14x + 10.29$

$\frac{dy}{dx} = \frac{-14x + 2y}{-2x + 12y} \quad \frac{32}{28}$

16. ***Area*** The radius, *r*, of a circle is decreasing at a rate of 4 centimeters per minute.

$y - 2 = \frac{32}{28}(x + 2)$

Find the rate of change of area, *A*, when the radius is 5

A) $\frac{dA}{dt} = -20\pi$

B) $\frac{dA}{dt} = -200\pi$

C) $\frac{dA}{dt} = 200\pi$

D) $\frac{dA}{dt} = -40\pi$

E) $\frac{dA}{dt} = 40\pi$

$A = \pi r^2$

$\frac{dA}{dt} = 2\pi r \cdot \frac{dr}{dt}$

$\frac{dA}{dt} = 2\pi(5)(-4) = 40\pi \text{ cm}^2/\text{min}$

Chapter 3 Review

1. Locate the absolute extrema of the function $f(x) = 2x^2 - 4x - 2$ on the closed interval $[-2,2]$.

$f'(x) = 4x - 4 = 0$

$4x = 4$

$x = 1$

$f(1) = -4$ min

$f(-2) = 14$ max

$f(2) = -2$

A) ~~No absolute max; Absolute min: $f(-2) = 14$~~
B) Absolute max: $f(1) = -4$; Absolute min: $f(-2) = 14$
C) ~~Absolute max: $f(-2) = 14$; No absolute min~~
D) $\boxed{\text{Absolute max: } f(-2) = 14 \text{ ; Absolute min: } f(1) = -4}$
E) ~~No absolute max or min~~

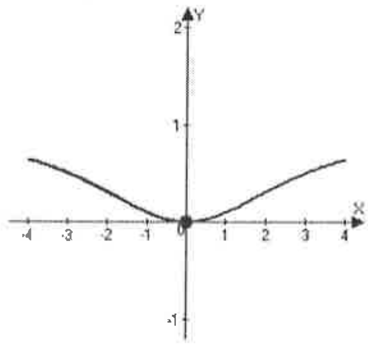
2. Determine whether the Mean Value Theorem can be applied to the function $f(x) = x^2$ on the closed interval [4,8]. If the Mean Value Theorem can be applied, find all numbers c in the open interval (4,8) such that $f'(c) = \frac{f(8) - f(4)}{8-4}$.

$f'(c) = \frac{64-16}{4} = \frac{48}{4} = 12$

$12 = 2x$ $x = 6$

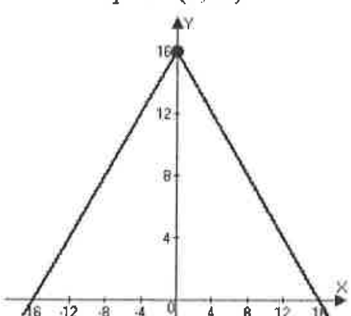
A) MVT applies; 5
B) MVT applies; 7
C) MVT applies; 6
D) MVT applies; 8
E) MVT applies; 4

3. Find the value of the derivative (if it exists) of the function $f(x) = \frac{x^2}{x^2+9}$ at the extremum point (0,0).



A) 0 B) 1 C) -1 D) $\frac{1}{4}$ E) $-\frac{1}{4}$

4. Find the value of the derivative (if it exists) of the function $f(x) = 16 - |x|$ at the extremum point (0,16).



A) 0 B) Does not exist C) 16 D) −16 E) None of the above

5. Identify the open intervals where the function $f(x) = 2x^2 + x - 3$ is increasing or decreasing.

A) Decreasing: $\left(-\infty, -\frac{1}{4}\right)$; Increasing: $\left(-\frac{1}{4}, \infty\right)$

B) Increasing: $\left(-\infty, -\frac{1}{4}\right)$; Decreasing: $\left(-\frac{1}{4}, \infty\right)$

C) Increasing on $(-\infty, \infty)$

D) Decreasing on $(-\infty, \infty)$

E) None of the above

$f'(x) = 4x + 1 = 0$

$4x = -1$

$x = -\frac{1}{4}$

$(-\infty, -\frac{1}{4})$ -1 $\downarrow$

$(-\frac{1}{4}, \infty)$ 0 $\uparrow$

6. For the function $f(x) = 2x^3 - 21x^2 + 5$:

(a) Find the critical numbers of f (if any);
(b) Find the open intervals where the function is increasing or decreasing; and
(c) Apply the First Derivative Test to identify all relative extrema.

Then use a graphing utility to confirm your results.

A) (a) $x = 0, 7$
(b) Increasing: $(-\infty, 0) \cup (7, \infty)$; Decreasing: $(0, 7)$
(c) Relative max: $f(0) = 5$; Relative min: $f(7) = -338$

B) (a) $x = 0, 7$
(b) Decreasing: $(-\infty, 0) \cup (7, \infty)$; Increasing: $(0, 7)$
(c) Relative min: $f(0) = 5$; Relative max: $f(7) = -338$

C) (a) $x = 0, 1$
(b) Increasing: $(-\infty, 0) \cup (1, \infty)$; Decreasing: $(0, 1)$
(c) Relative max: $f(0) = 5$; Relative min: $f(1) = -14$

D) (a) $x = 0, 1$
(b) Decreasing: $(-\infty, 0) \cup (1, \infty)$; Increasing: $(0, 1)$
(c) Relative min: $f(0) = 5$; Relative max: $f(1) = -14$

$f'(x) = 6x^2 - 42x = 0$

$6x(x - 7) = 0$

$x = 0 \quad x = 7$

$(-\infty, 0)$ $\uparrow$ 0 - max

$(0, 7)$ $\downarrow$

$(7, \infty)$ $\uparrow$ 7 - min