

Calculus
Chapter 5 Review Exercises

Name _____

Find the derivative of the function.

1. $y = \ln \sqrt{x}$

2. $g(x) = x^2 e^x$

3. $g(x) = 3^{x-1}$

4. $f(x) = (4e)^x$

5. $f(x) = \ln \frac{e^x}{1+e^x}$

6. $y = \sqrt{e^{2x} + e^{-2x}}$

7. $y = \ln \frac{x(x-1)}{x-2}$

8. $y = x\sqrt{\ln x}$

9. $y = \ln \left[x(x^2 - 2)^{\frac{2}{3}} \right]$

10. $y = x^{2x+1}$

11. $y = x(4^{-x})$

12. $h(z) = e^{\frac{z^2}{2}}$

13. $h(x) = \frac{x^2}{e^x}$

14. $y = \log_3 \sqrt{1-x}$

15. $y = 3e^{\frac{3}{t}}$

16. $y = \log_5 \frac{x}{x-1}$

Find or evaluate the integral.

17. $\int \frac{1}{7x-2} dx$

18. $\int_0^1 x e^{-3x^2} dx$

19. $\int (x+1)5^{(x+1)^2} dx$

20. $\int \frac{x}{x^2-1} dx$

21. $\int \frac{\sin x}{1+\cos x} dx$

22. $\int_{\frac{1}{2}}^2 \frac{e^{\frac{1}{x}}}{x^2} dx$

23. $\int \frac{e^{4x} - e^{2x} + 1}{e^x} dx$

24. $\int \frac{\ln \sqrt{x}}{x} dx$

25. $\int_1^e \frac{\ln x}{x} dx$

26. $\int \frac{2^{\frac{1}{t}}}{t^2} dt$

27. $\int x e^{1-x^2} dx$

28. $\int_1^4 \frac{x+1}{x} dx$

29. $\int \frac{e^{2x} - e^{-2x}}{e^{2x} + e^{-2x}} dx$

30. $\int_1^3 \frac{e^x}{e^x - 1} dx$

31. $\int x^2 e^{x^3+1} dx$

32. Find $\frac{dy}{dx}$: $y \ln x + y^2 = 0$

Solve the equation for x.

33. $\ln \sqrt{x+1} = 2$

34. $\ln x + \ln(x-3) = 0$

Find the equation of the tangent line at the given point.

35. $f(x) = \ln(e^{-x^2})$ (2, -4)

36. $f(\theta) = \frac{1}{2} e^{\sin 2\theta}$ $\left(0, \frac{1}{2}\right)$

37. $y = \ln(2+x) + \frac{2}{2+x}$ (-1, 2)

Find the area of the region bounded by the graphs of the equations.

38. $y = x e^{-x^2}$, $y = 0$, $x = 0$, $x = 4$

39. $y = 2e^{-x}$, $y = 0$, $x = 0$, $x = 2$

Chapter 5 Review Exercises

① $y = \ln \sqrt{x} = \frac{1}{2} \ln x$

$$y' = \frac{1}{2x}$$

② $g(x) = x^2 e^x$

$$g'(x) = x^2 e^x + 2x e^x$$

③ $g(x) = 3^{x-1}$

$$g'(x) = (\ln 3) 3^{x-1}$$

④ $f(x) = (4e)^x = 4^x e^x$

$$f'(x) = 4^x e^x + e^x (\ln 4) (4^x)$$

⑤ $f(x) = \ln \frac{e^x}{1+e^x} = x \ln e - \ln(1+e^x)$

$$f'(x) = 1 - \frac{e^x}{1+e^x}$$

⑥ $y = \sqrt{e^{2x} + e^{-2x}} = (e^{2x} + e^{-2x})^{1/2}$

$$y' = \frac{1}{2} (e^{2x} + e^{-2x})^{-1/2} (2e^{2x} - 2e^{-2x})$$

$$y' = \frac{e^{2x} - e^{-2x}}{\sqrt{e^{2x} + e^{-2x}}}$$

⑦ $y = \ln \frac{x(x-1)}{x-2} = \ln x + \ln(x-1) - \ln(x-2)$

$$y' = \frac{1}{x} + \frac{1}{x-1} - \frac{1}{x-2}$$

⑧ $y = x \sqrt{\ln x}$

$$y' = \frac{1}{2} x (\ln x)^{-1/2} \left(\frac{1}{x}\right) + \sqrt{\ln x}$$

$$y' = \frac{1}{2\sqrt{\ln x}} + \sqrt{\ln x}$$

⑨ $y = \ln [x(x^2-2)^{2/3}] = \ln x + \frac{2}{3} \ln(x^2-2)$

$$y' = \frac{1}{x} + \frac{4x}{3(x^2-2)}$$

⑩ $y = x^{2x+1}$

$$\ln y = (2x+1) \ln x$$

$$\frac{y'}{y} = (2x+1) \frac{1}{x} + 2 \ln x$$

$$y' = y \left(\frac{2x+1}{x} + 2 \ln x \right)$$

$$y' = x^{2x+1} \left(\frac{2x+1}{x} + 2 \ln x \right)$$

* ⑪ $y = x(4^{-x})$

$$y' = -x (\ln 4) (4^{-x}) + 4^{-x}$$

⑫ $n(z) = e^{-\frac{z^2}{2}}$

$$n'(z) = -z e^{-\frac{z^2}{2}}$$

⑬ $h(x) = \frac{x^2}{e^x}$

$$h'(x) = \frac{2x e^x - x^2 e^x}{(e^x)^2}$$

$$h'(x) = \frac{2x - x^2}{e^x}$$

⑭ $y = \log_3 \sqrt{1-x} = \frac{1}{2} \log_3 (1-x)$

$$y' = \frac{-1}{2(\ln 3)(1-x)}$$

$$(15) \quad y = 3e^{-3/t}$$

$$y' = \frac{9e^{-3/t}}{t^2}$$

$$(16) \quad y = \log_5 \frac{x}{x-1} = \log_5 x - \log_5(x-1)$$

$$y' = \frac{1}{(\ln 5)(x)} + \frac{1}{(\ln 5)(x-1)}$$

$$(17) \quad \int \frac{(1)^7}{7x-2} dx \quad u=7x-2, \quad du=7dx, \quad \frac{1}{7} \int \frac{1}{u} du$$

$$= \frac{1}{7} \ln |7x-2| + C$$

$$(18) \quad \int_0^1 \frac{1}{6} e^{-3x^2} dx = \frac{-1}{6} \int e^u du$$

$$u = -3x^2, \quad du = -6x dx$$

$$= \frac{-1}{6} \left[e^{-3x^2} \right]_0^1$$

$$= \frac{-1}{6} \left[\frac{1}{e^3} - 1 \right] = \left[\frac{-1}{6e^3} + \frac{1}{6} \right] \approx 0.158$$

$$(19) \quad \int \frac{1}{2} (x+1) 5^{(x+1)^2} dx \quad u=(x+1)^2, \quad du=2(x+1)dx$$

$$= \frac{1}{2} \int 5^u du = \frac{1}{2 \ln 5} (u) + C$$

$$= \frac{1}{2 \ln 5} (5^{(x+1)^2}) + C$$

$$(20) \quad \int \frac{2x}{x^2-1} dx = \int \frac{1}{u} du$$

$$u = x^2-1, \quad du = 2x dx$$

$$= \frac{1}{2} \ln |x^2-1| + C$$

$$(21) \quad \int \frac{-\sin x}{1+\cos x} dx \quad u=1+\cos x, \quad du=-\sin x dx$$

$$= \int \frac{1}{u} du = \ln |1+\cos x| + C$$

$$(22) \quad \int_{\frac{1}{2}}^2 \frac{e^{1/x}}{x^2} dx \quad u = \frac{1}{x}, \quad du = -\frac{1}{x^2} dx$$

$$= \int_{\frac{1}{2}}^2 e^u du = - \left[e^{1/x} \right]_{\frac{1}{2}}^2 = - \left(e^{1/2} - e^2 \right) = 5.740$$

$$(23) \quad \int \frac{e^{3x} - e^{2x} + 1}{e^x} dx = \int (e^{3x} - e^x + e^{-x}) dx$$

$$= \frac{1}{3} e^{3x} - e^x - \frac{1}{e^x} + C$$

$$(24) \quad \int \frac{\ln x}{x} dx \quad u = \ln x, \quad du = \frac{1}{x} dx$$

$$= \int u du = \frac{1}{2} \cdot \frac{1}{2} u^2 + C$$

$$= \frac{1}{4} (\ln x)^2 + C$$

$$(25) \quad \int_1^e \frac{\ln x}{x} dx \quad u = \ln x, \quad du = \frac{1}{x} dx$$

$$= \int_1^e u du = \frac{1}{2} [u^2]_1^e$$

$$= \frac{1}{2} [(\ln x)^2]_1^e = \frac{1}{2} \ln e - \frac{1}{2} \ln 1$$

$$= \frac{1}{2}$$

$$(26) \quad \int \frac{2^{-1/t}}{t^2} dt \quad u = -1/t, \quad du = 1/t^2 dt$$

$$= \int 2^u du = \frac{1}{\ln 2} (2^{-1/t}) + C$$

$$(27) \quad \int x e^{1-x^2} dx \quad u = 1-x^2, \quad du = -2x dx$$

$$= \frac{-1}{2} \int e^u du = -\frac{1}{2} e^{1-x^2} + C$$

$$(29) \quad \int \frac{e^{2x} - e^{-2x}}{e^{2x} + e^{-2x}} dx \quad u = e^{2x} + e^{-2x}, \quad du = 2(e^{2x} - e^{-2x}) dx$$

$$= \frac{1}{2} \int \frac{1}{u} du = \frac{1}{2} \ln |e^{2x} + e^{-2x}| + C$$

$$\begin{aligned} \textcircled{30} \int_1^3 \frac{e^x}{e^x-1} dx & \quad u = e^x - 1 \\ & \quad du = e^x dx \\ \int_1^3 \frac{1}{u} du & = [\ln|e^x-1|]_1^3 \\ & = \ln(e^3-1) - \ln(e-1) \\ & = \ln\left(\frac{e^3-1}{e-1}\right) \approx 2.408 \end{aligned}$$

$$\begin{aligned} \textcircled{31} \int \frac{1}{3} 3x^2 e^{x^3+1} dx & \quad u = x^3 + 1 \\ & \quad du = 3x^2 dx \\ \frac{1}{3} \int e^u du & = \boxed{\frac{1}{3} e^{x^3+1} + C} \end{aligned}$$

$$\begin{aligned} \textcircled{32} y \ln x + y^2 & = 0 \\ y\left(\frac{1}{x}\right) + \frac{dy}{dx}(\ln x) + 2y\left(\frac{dy}{dx}\right) & = 0 \\ \frac{dy}{dx}(\ln x + 2y) & = -\frac{y}{x} \\ \boxed{\frac{dy}{dx} = \frac{-y}{x(\ln x + 2y)}} \end{aligned}$$

$$\begin{aligned} \textcircled{33} \ln \sqrt{x+1} & = 2 \\ e^2 & = \sqrt{x+1} \\ e^4 & = x+1 \\ \boxed{x = e^4 - 1 \approx 53.598} \end{aligned}$$

$$\begin{aligned} \textcircled{34} \ln x + \ln(x-3) & = 0 \\ \ln(x^2-3x) & = 0 \\ x^2-3x & = 1 \quad x^2-3x-1=0 \\ x = \frac{3 \pm \sqrt{13}}{2} & \quad \boxed{x = \frac{3 + \sqrt{13}}{2}} \end{aligned}$$

$$\begin{aligned} \textcircled{35} y & = \frac{1}{2}x - 3 \\ x & = \frac{1}{2}y - 3 \\ x+3 & = \frac{1}{2}y \\ \boxed{2(x+3) = y^{-1}} \end{aligned}$$

$$\begin{aligned} \textcircled{36} f(x) & = x^2 - 5, \quad x \geq 0 \\ x & = y^2 - 5 \\ x+5 & = y^2 \\ \pm \sqrt{x+5} & = y^{-1}, \quad x \geq 0 \\ \boxed{y^{-1} = \sqrt{x+5}} \end{aligned}$$

$$\begin{aligned} \textcircled{37} f(x) & = \sqrt[3]{x+1} \\ x^3 & = y+1 \\ \boxed{x^3 - 1 = y^{-1}} \end{aligned}$$

$$\begin{aligned} \textcircled{35} f(x) & = \ln(e^{-x^2}) = -x^2 \ln e = -x^2 \\ f'(x) & = -2x \quad @ (2, -4) \\ f'(2) & = -2(2) = -4 \\ \boxed{y+4 = -4(x-2)} \end{aligned}$$

$$\begin{aligned} \textcircled{36} f(\theta) & = \frac{1}{2} e^{\sin 2\theta} \quad \left(0, \frac{1}{2}\right) \\ f'(\theta) & = e^{\sin 2\theta} \cdot \cos 2\theta \\ f'(0) & = e^0 \cdot \cos(0) = 1 \\ y^{-\frac{1}{2}} & = x \\ \boxed{y = x + \frac{1}{2}} \end{aligned}$$

$$\begin{aligned} (37) \quad y &= \ln(2+x) + \frac{2}{2+x} \quad (-1, 2) \\ y' &= \frac{1}{2+x} - \frac{2}{(2+x)^2} \quad y'(-1) = 1 - 2 = -1 \\ y - 2 &= -1(x+1) \end{aligned}$$

$$\begin{aligned} (38) \quad y &= x e^{-x^2} \\ \frac{1}{2} \int_0^4 2x e^{-x^2} dx & \quad u = -x^2 \quad du = -2x dx \\ \frac{1}{2} [e^u du] &= \frac{1}{2} [e^{-x^2}]_0^4 \\ \frac{1}{2} [e^{-16} - 1] &\approx 0.500 \end{aligned}$$

$$\begin{aligned} (39) \quad y &= 2e^{-x} \\ -2 \int_0^2 2e^{-x} dx & \quad u = -x \quad du = -1 dx \\ &= -2 [e^{-x}]_0^2 \\ &= -2 \left[\frac{1}{e^2} - 1 \right] \approx 1.729 \end{aligned}$$